Enhancement of a Constitutive Friction Law by Considering Plastic Smoothing of Rough Surfaces in Dependency on the Bulk Stresses

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ABSTRACT

The aim of this work is the improvement of a constitutive friction law which has been identified for the specific demands of sheet-bulk metal-forming. The constitutive friction law determines the friction shear stress in dependency on the plastic smoothing of the surface roughness, which also affects the real contact area. An experimental setup is introduced to determine the change of the surface roughness, which is influenced by the tensile bulk stress in the workpiece. In order to model the experimental results, a well-established half-space model is presented and improved to consider strain hardening. The half-space model is used to determine the change of the contact area as a function of the tensile bulk stress. This interdependency is also reflected by the proposed constitutive friction law.

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1. INTRODUCTION

Friction between the die and the workpiece plays a vital role in forming processes. It impacts wear and lifetime of the tool, the flow of the deformed material and the required process forces. Friction is influenced by many factors, for example temperature, sliding velocity, flow stress, and the surface roughness. Although friction has been widely studied, the big amount of factors influencing friction still inhibits a general mathematical formulation of the friction interaction. This is also reflected in the broad range of different friction laws, which are available for the simulation of forming processes. Forming processes are commonly classified into sheet metal processes and bulk forming processes. The Coulomb friction model is an accepted model for the simulation of sheet metal forming, which mostly occurs with low to moderate contact loads. The Coulomb friction model describes the friction shear stress as a function of the normal stress and the friction coefficient. Furthermore, Westeneng proposed a model in [1] that focuses on the boundary lubrication and ploughing regime in order to provide an advanced model especially for
deep drawing. In a similar way, Hol developed a boundary lubrication friction model that relies on the surface deformation of the contacting surfaces [2]. The latter model is designed for the particular needs of automotive sheet metal forming.

On the other hand, bulk forming processes, which take place with very high contact loads, are commonly simulated with the friction model of Tresca. This model estimates the friction stress by using the shear strength of the weaker material in contact and the friction factor. Alternatively, a plastic wave friction model with the ability to take into account mixed lubrication was presented by Vidal-Sallé et al. in [3].

However, forming processes which show characteristics of both sheet metal forming and bulk metal forming make the choice of an applicable friction law difficult. A prime example for this is sheet-bulk metal forming, as bulk forming processes are applied onto metal sheets [4]. The products of sheet-bulk metal forming are lightweight high-precision components with functional integration, which grant this process an outstanding role. Sheet-bulk metal forming comes with both low and high contact loads. Therefore, the friction models of Shaw [5, 6], or Wanheim and Bay [7, 8] are more suitable, since these laws provide a smooth transition between Coulomb’s friction model and Tresca’s one. This characteristic is also important for the constitutive friction law (CFL) which is presented in [9].

The CFL is mathematically formulated as:

\[ \tau = m \cdot k \cdot \alpha, \]

where \( \tau \) is the friction shear stress, \( k \) is the shear yield strength, and \( \alpha \) denotes the contact ratio between the real contact area \( A_r \) and the apparent area \( A_0 \). The friction factor \( m \) has to be measured experimentally, for instance by ring upsetting [10]. Due to the roughness of technical surfaces, \( A_0 \) is only in contact with its surface asperities. The area which is provided by these asperities is \( A_r \). The parameter \( \alpha \) is a relevant indicator as contact loads are only transferable in \( A_r \), or by hydrodynamic effects.

The CFL distinguishes between initial contact and un-/reloading. For initial contact \( \alpha \) is identified similar to the findings of Shaw as:

\[ \alpha = \alpha_2 \left( \frac{b_2 \cdot p_{\text{mean}}}{\alpha(p_h) \cdot H} \right)^{a_2}, \]

where \( p_{\text{mean}} \) is the contact pressure acting on \( A_0 \). \( H \) is the surface hardness and \( \alpha_2 \) and \( b_2 \) are numerically identified parameters.

As the CFL is the product of the shear strength and the contact ratio, it can be interpreted as Tresca’s friction law taking effect only in \( A_r \). Due to the plastic deformation of the surface, \( A_r \) increases linearly with \( p_{\text{mean}} \) for low to moderate contact loads. Since \( A_r \) cannot surpass \( A_0 \) \( \alpha \) approaches 100% for large \( p_{\text{mean}} \). For low contact loads the CFL shows a behavior similar to Coulomb’s friction law. Yet, for high contact loads, the CFL is related to Tresca’s law of friction. A qualitative comparison between Tresca’s law, Coulomb’s law and the CFL is shown in Fig. 1.

![Fig. 1. Qualitative comparison of friction models.](image)

As sheet-bulk metal forming can occur incrementally, the CFL is able to take into account un- and reloading. Un-/reloading is mostly elastic, since the plastic surface deformation already occurred at the initial contact. For un-/reloading the CFL is also expressed by Eq. (1), but with an altered \( \alpha \):

\[ \alpha = \left( \frac{b_2 \cdot p_{\text{mean}}}{\alpha(p_h) \cdot H} \right)^{a_2} \cdot \alpha(p_h). \]

In Eq. (3) the parameters \( a_2 \) and \( b_2 \) are identified numerically. Additionally, \( \alpha \) for un-/reloading depends on the historic contact load \( p_h \), which is the maximum contact load of the previous contacts. The parameter \( \alpha(p_h) \) takes into account the plastic surface smoothing which arose in a previous contact step with \( p_h \). This leads to a real contact area for un-/reloading which is larger than the real contact area at initial contact. Fig. 2
shows the progression of $\tau$ for initial contact and for un-/reloading.

![Fig. 2. Friction law with un- and reloading.](image)

Certainly, sheet-bulk metal forming is also characterized by local highly varying two-dimensional and three-dimensional stress and strain states in the bulk material. This condition is not incorporated by the CFL.

In order to identify $\alpha$ in dependency on the bulk stress, the elastic-plastic half-space model which was used in [9] has to be enhanced. Sec. 2 describes this half-space model and provides, additionally, an implementation that considers the surface smoothing as a function of the strain hardening and bulk stresses. That followed, an experimental set-up is described in Sec. 3, which enables the determination of the surface smoothing in dependency on the tensile stress in the bulk material. Sec. 4 describes the subsequent validation and calibration of the half-space model using the experimental results. In Sec. 4 the newly developed CFL is described.

### 2. NUMERICAL MODEL

#### 2.1 Elastic Half-Space Model

The contact between rough surfaces is a multi-scale problem. Hence, the investigated surfaces have to be large enough to be representative. Moreover, the surface discretization has to be very fine. The finite element method is a viable method for such a challenge due to its universality, as shown in [[11], [12], [13]].

However, a half-space model is preferred because such a model only needs to solve the contact problem at the two-dimensional surface boundary [14]. Hereafter, the complexity of the half-space model is much lower than the one of the finite element method.

The elastic half-space model is based on the work of Tian and Bhushan [15], which is a well-established method for the contact simulation between two linear elastic bodies. The surface of each body is discretized into $N_x \times N_y = M$ elements on a regular grid with element size $2a \times 2b$. Fig. 3 illustrates an example with $N_x \times N_y = 4 \times 3$ elements for the length $L_x$ in $x$-direction and $L_y$ in $y$-direction.

![Fig. 3. Top view of discretized surface.](image)

The surface deformation is defined by the following three equations:

$$g_k = h_{\max} - h_k + u_k - u_0,$$  \hspace{1cm} (4)

$$\sum_{k=1}^{M} p_k \cdot 2a \cdot 2b = p_{\text{mean}} \cdot L_x \cdot L_y,$$  \hspace{1cm} (5)

$$u_k = \sum_{l=1}^{M} C_{kl} \cdot p_l.$$  \hspace{1cm} (6)

Eq. (4) determines the gap $g_k$ of the surface element $k$ in dependency on the maximum height $h_{\max}$ of the rough surface, the local height $h_k$ of element $k$, the local surface deformation $u_k$ and the global body approach $u_0$. The interrelationship between the variables is shown in Fig. 4.

The condition of equilibrium is defined by Eq. (5). The latter shows that the mean contact pressure $p_{\text{mean}}$ acting on the whole discretized area $L_x \times L_y$ has to be in balance with the sum of the local contact pressure $p_k$. The contact pressure $p_k$ is nonzero only for the elements $l$ that are in actual contact.
The surface deformation at \( k \) due to the load at element \( l \) is described by Eq. (6), which is based on the analytic Boussinesq solution [16]. The influence coefficient matrix \( C_{kl} \) is:

\[
C_{kl} = \frac{1}{\pi E'} \int_{a}^{b} \int_{\alpha}^{\beta} \frac{d\xi d\eta}{\rho},
\]

where the effective elastic modulus \( E' \) is defined as:

\[
\frac{1}{E'} = \frac{1}{E_1} + \frac{1}{E_2},
\]

with \( E_i \) and \( v_i \) as Young's moduli and Poisson's ratios of the contacting bodies, respectively. In addition, the distance between the load point \( l \) and the field point \( k \) is given by:

\[
\rho = \sqrt{(x_k - x_l - \xi)^2 + (y_k - y_l - \eta)^2}.
\]

Love presented in [17] a solution to Eq. (7):

\[
C_{kl} = \frac{1}{\pi E'} \left[ x_1 \ln \left( \frac{y_1 + \sqrt{x_1^2 + y_1^2}}{y_2 + \sqrt{x_1^2 + y_2^2}} \right) + x_2 \ln \left( \frac{y_2 + \sqrt{x_2^2 + y_2^2}}{y_1 + \sqrt{x_2^2 + y_1^2}} \right) + y_1 \ln \left( \frac{x_1 + \sqrt{x_1^2 + y_1^2}}{x_2 + \sqrt{x_1^2 + y_2^2}} \right) + y_2 \ln \left( \frac{x_2 + \sqrt{x_2^2 + y_2^2}}{x_1 + \sqrt{x_2^2 + y_1^2}} \right) \right],
\]

with

\[
x_1 = x + a, \quad x_2 = x - a, \quad (11)
\]

and

\[
y_1 = y + b, \quad y_2 = y - b. \quad (12)
\]

The contact occurs neither with penetration nor with adhesion. Therefore, the system which determines the surface deformation, i.e., Eqs. (4) to (6), has to be solved in conjunction with the complementarity conditions, which are given by the two equations:

\[
g_k = 0, \quad p_k > 0, \quad k \in I_c \quad (13)
\]

and

\[
g_k > 0, \quad p_k = 0, \quad k \not\in I_c, \quad (14)
\]

where \( I_c \) is a set including all elements that are in contact.

As it is described in detail by Allwood in [18], the nonlinear contact problem for elastic contact is efficiently solved for the target \( p_{\text{mean}} \) by using an active set strategy in combination with the conjugate gradient method.

### 2.2 The Elastic-Plastic Half-Space Model With Work Hardening

The elastic contact model is extended to an elastic-plastic contact model by limiting the local contact pressure \( p_k \) by the surface hardness of the softer contacting material \( H \), i.e.,

\[
p_k \leq H, \quad l = 1, \ldots, M. \quad (15)
\]

According to Bowden and Tabor [19], \( H \) depends on the yield strength \( \sigma_y \) with:

\[
H \approx 2.8 \cdot \sigma_y. \quad (16)
\]

In addition, the surface displacement \( u_k \) is divided additively into an elastic surface displacement \( u_{el,k} \) and a plastic surface displacement \( u_{pl,k} \),

\[
u_k = u_{el,k} + u_{pl,k}. \quad (17)
\]

The determination of the plastic deformation for \( p_{\text{mean}} \) is performed after the evaluation of the solely elastic solution for the same \( p_{\text{mean}} \). As it is described in detail in [20], contact elements with a local contact pressure that surpasses the local hardness \( H_k \) are added to a ‘plastic set’, in which \( p_k \) is defined as:

\[
p_k = H_k. \quad (18)
\]

That followed, the deformation \( u_{pl,k} \), due to the elements in the plastic set, as well as its resulting load \( p_{pl} \) are determined. The gap equation (4) is reformulated by:

\[
g_k = h_{\text{max}} - h_k + u_{el,k} + u_{pl,k} - u_0, \quad (19)
\]

which has to be solved for an updated \( p_{\text{mean}} \) subtracted by \( p_{pl} \). The plastic surface
displacement has to be determined iteratively, until the elastic solution and Eqs. (17) to (19) are converged.

The hitherto described model considers ideal elastic-plastic material behavior with a constant hardness. The next extension of the contact model aims to model the plasticity with the Hockett-Sherby hardening law [21]. This nonlinear hardening function describes \( \sigma_y \) in dependency on the true strain \( \varepsilon \) as:

\[
\sigma_y = \sigma_\infty + (\sigma_0 - \sigma_\infty) \cdot \exp(a_{HS} \cdot \varepsilon^{b_{HS}}),
\]

where the initial and infinite yield strength are denoted by \( \sigma_0 \) and \( \sigma_\infty \), respectively, and the curvature multipliers are denoted by \( a_{HS} \) and \( b_{HS} \).

The reference height \( \lambda \) is necessary to determine \( \varepsilon \) of a surface element, since the actual length of an element in a half-space model is infinite. Inspired by Hol [2] the true strain \( \varepsilon_k \) of element \( k \) is defined as:

\[
\varepsilon_k = \ln\left(1 + \frac{u_{pl,k}}{\lambda}\right),
\]

where the characteristic length \( \lambda \) is assumed to be equal for each surface element.

Further, the contact model also considers volume conservation based on the experimental results of Pullen and Williamson [22]. As shown in Fig. 5, the volume which is displaced due to the plastic deformation of the surface asperities is evenly added to the surface valleys which are not in contact.

![Fig. 5. Definition of \( \lambda \) and example for volume conservation.](image)

This contact model solves the contact problem in dependency on the interfacial pressure. However, it does not take into account the stress and strain condition in the bulk. Three-dimensional contact models which are based on the Boussinesq solution and take into account such stress and strain rates have been proposed for example by Jacq et al. [23] for rolling contact, and by Nélias et al. [24] in combination with a von Mises yield criterion. According to Johnson [25], there exist three different contact zones, this being the elastic zone, the elastic-plastic zone, and the full plastic zone. Contact models such as the ones presented in [23, 24] are especially suited for the elastic-plastic regime [26].

As the contact loads are very large in the following experimental setup, the focus of this study is the fully plastic regime. This allows to apply the presented contact model and brings the advantage of reduced numerical effort. The disadvantage of the simplified contact model is the inability to directly take into account bulk stresses. Therefore, the surface hardness is also expressed in dependency on the bulk stress:

\[
\left(\frac{p_k}{\beta}\right)^2 + \left(\frac{\sigma_x}{\gamma}\right)^2 + \frac{p_k \cdot \sigma_x}{\beta \cdot \gamma} \leq \sigma_{y,k}^2.
\]

The latter is a relation motivated by the Tabor equation [27]. The tensile stress is denoted by \( \sigma_x \) and the parameters \( \beta \) and \( \gamma \) are factors which are identified in Sec. 4.2.

### 3. EXPERIMENTAL SETUP

A test rig was developed to determine the roughness smoothing in dependency on \( p_{\text{mean}} \) and the bulk stress \( \sigma_x \). It is shown in Fig. 6.

![Fig. 6. Experimental setup.](image)

The workpiece material, with the dimensions 300 mm x 30 mm and an initial thickness of 2 mm, is clamped on both sides. The bulk stress \( \sigma_x \) is applied on the workpiece material in x-direction. The values of \( \sigma_x \) are proportional to \( \sigma_0 \) of the investigated material and varied from 0 to 1.0\( \sigma_0 \) in steps of 0.25\( \sigma_0 \). These values guarantee a broad investigation of the pre-strain on the
surface smoothing. The bulk stress is superimposed by \( p_{\text{mean}} \), which is applied due to the movement of the upper punch in z-direction. The chosen values for \( p_{\text{mean}} \) depend on the material. The experiments were conducted using DC04 and DP600 with electrical discharge texturing (EDT) surfaces. For DC04, \( p_{\text{mean}} \) took the values 100, 250, 500 and 750 N/mm². As the yield strength of DP600 is more than twice as high as that of DC04, the value 1000 N/mm² was also included in the experiment with DP600. The analyses are realized without using a lubricant between punch and workpiece. By doing this, the influence of lubrication, like viscosity, is avoided. To ensure a dry contact condition, the workpiece surface was cleaned with acetone. The surface texture before and after the experiments were measured using the confocal laser scanning microscope Keyence VK-X200. An area of 2.63 mm x 2.81 mm was measured to guarantee stable roughness values. The measurements were performed with an objective lens with an optical magnification of 20x.

4. RESULTS

4.1 Experimental Results

In order to validate the numerical results, the surface smoothing as a function of the nominal contact load and the bulk stress was determined experimentally. Exemplary topographies are illustrated, to investigate the smoothing qualitatively. The surface smoothing is quantified by means of \( R_a \). This value is the \( R_a \) value after forming referenced to the initial average roughness value of the DC04 blanks which amounts to 0.94 ± 0.01 µm. Thus, the smoothening is investigated using the change of the average roughness value.

Fig. 7 shows the impact of using different \( p_{\text{mean}} \) on the topography of a DC04 blank without applying an additional bulk stress. An increase of the nominal contact load from 250 N/mm² to 750 N/mm² leads to a significant smoothing of the surface. The higher the load the higher the flattening is. This qualitative correlation can be verified by comparing the referenced average roughness values, see Fig. 8. A high value of \( p_{\text{mean}} \) results in higher smoothing of the surface. A significant decrease of \( R_a \) can be observed for the load values 250 N/mm², 500 and 750 N/mm². Yet, a contact load of 100 N/mm² leads to only a slight flattening of the surface. The nominal contact load of 100 N/mm² is not high enough to significantly plastically deform the material. All in all, it can be stated that the application of just a nominal contact load and thus a uniaxial stress state leads to a significant height reduction of the roughness peaks and, thus, to an increased smoothing of the surface. Additionally, the dimension of the height reduction is highly influenced by the amount of the applied load.

The combined application of a nominal contact load and a bulk stress additionally influences the surface smoothing. Thus, the flattening of the surface is strengthened due to the application of an additional bulk stress. However, the influence of the applied bulk stress is not as distinctive as the one of the nominal contact load.
An increase of the applied bulk stress leads to a slight decrease of the $R_{a,ref}$ values. For a nominal contact load of 500 N/mm² the $R_{a,ref}$ value constantly decreases from $0.44 \pm 0.01 \mu m$ for $0 \cdot \sigma_0$ to $0.36 \pm 0.01 \mu m$ for $1 \cdot \sigma_0$. Thus, the flattening of the roughness peaks increases with increasing bulk stress values due to the two-dimensional stress state. The same tendencies are valid for the other investigated nominal contact loads.

Additionally, Fig. 9 shows the topographies of DC04 blanks for a nominal contact load of 250 N/mm² with varying bulk stresses. The topographies verify the results of the quantitative investigation using $R_{a,ref}$. It can be observed that an increase of the bulk stress leads to a more flat profile of the roughness peaks.

Fig. 10 depicts $R_{a,ref}$ for DP600. It can be observed that for $p_{mean} = 100$ and 250 N/mm² and all bulk stress values no significant decrease of $R_{a,ref}$ is provided. It should be noted that DP600 has a higher yield strength than DC04. This is clarified by Table 1 and Table 2, which show the Hockett-Sherby parameters for both materials. Consequently, as $p_{mean} = 100$ and 250 N/mm² are low, no significant flattening of the roughness peaks are observed. For the higher pressure values of 500, 750 and 1000 N/mm² and all investigated bulk stress values a decrease of roughness with increasing load and bulk stress can be observed. Thus, for high contact loads and bulk stresses, the experimental results obtained for DP600 are similar to those obtained for DC04.

Comparing DC04 and DP600, differences are given by the degree of change. The $R_{a,ref}$ values for DC04 are lower than for DP600. Thus, the surface smoothing of DC04 is higher than that of DP600. The $R_{a,ref}$ value for a nominal contact load of 500 N/mm² combined with a bulk stress of $0 \cdot \sigma_0$ exemplary amounts to $0.44 \pm 0.01$ for DC04. The $R_{a,ref}$ for DP600 has a value of $0.88 \pm 0.03$. Thus, the reduction of roughness for DC04 is twice as high than the one for DP600. This can be explained by the different material properties. The initial yield strength of DC04 amounts to 185 N/mm². The value for DP600 is much higher and amounts to 385 N/mm². According to Eq. (16), the hardness is proportional to the yield strength. Thus, smoothing of the surface is strongly influenced by the material properties.

All in all it can be stated that the smoothing of the surface is significantly influenced by the applied bulk stress, the nominal contact load and the workpiece material. An application of a nominal contact load on pre-strained workpiece material leads to an increase of the surface smoothing in comparison to the application of just a nominal load. These experimentally determined values validate the results of the numerical investigation in Sec. 4.2.

### Table 1. Hockett-Sherby parameters for DC04.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$</td>
<td>185 N/mm²</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>577 N/mm²</td>
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<tr>
<td>$\alpha_{fs}$</td>
<td>2.1771</td>
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<tr>
<td>$b_{fs}$</td>
<td>0.6667</td>
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</table>
Table 2. Hockett-Sherby parameters for DP600.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$</td>
<td>385 N/mm²</td>
</tr>
<tr>
<td>$\sigma_\infty$</td>
<td>1050 N/mm²</td>
</tr>
<tr>
<td>$a_{HS}$</td>
<td>2.30</td>
</tr>
<tr>
<td>$b_{HS}$</td>
<td>0.6667</td>
</tr>
</tbody>
</table>

4.2 Numerical Results

The surfaces for the contact simulation were measured with a laser microscope Keyence VK-X105. The measured surfaces with $L_x \approx 2.63$ mm and $L_y \approx 2.81$ mm were obtained with $N_x \approx 3700$ and $N_y \approx 3900$ points. In order to perform the simulations in reasonable time, all surfaces were sampled down to $N_x = N_y = 256$ with $L_x = L_y = 2.56$ mm, which is similar to a low pass filtering. The transformation alters the surface roughness of the DC04 blank to $R_x = 1.089$ μm and $R_y = 1.329$ μm, and of the DP600 blank to $R_x = 0.907$ μm and $R_y = 1.107$ μm. The parameters of the tool surface are $R_x = 0.126$ μm and $R_y = 0.154$ μm. The tool surface is considered solely elastic, whereas the DC04 surface as well as the DP600 surface are modelled elastic-plastic with the Hockett-Sherby hardening law. Table 1 shows the parameters for DC04, which were identified by Schmaltz with the help of biaxial tests in [28]. Table 2 provides the parameters for DP600, which are based on the flow stress curve given in [29]. The remaining material parameters are $E = 210000$ N/mm² and $\nu = 0.30$ for every contact partner. The characteristic length was set to $\lambda = 6R_0$. In order to determine $\lambda$, $R_0$ is evaluated with the added height field of both tool and workpiece. The bulk stress $\sigma_b$ is set in dependency on the initial yield stress of the workpiece from 0 to $1.0\cdot\sigma_0$ in steps of $0.25\cdot\sigma_0$. The maximum nominal contact load constitutes 750 N/mm² and 1000 N/mm² for the DC04 surface and the DP600 surface, respectively. The normal load is always applied incrementally in steps of 10 N/mm². The parameter $\beta$ is set to 2.8, because Eq. (22) reduces to Eq. (16) for $\sigma_x = 0$ N/mm². Finally, the last parameter $\gamma$ is equal to 2.3.

The changes of $R_x$ and $R_y$ for DC04 are shown in Fig. 11 to Fig. 20. These figures depict a comparison between the results of the simulations and the experiments.

Both the simulations and the experiments approach $R_x \approx 0.19$ μm and $R_y \approx 0.25$ μm for high contact loads, which is in the magnitude of the tool surface. Furthermore, the surface roughness diminishes with increasing $\sigma_x$. The differences between the simulation and the experiments are negligible for $\sigma_x \leq 0.50\cdot\sigma_0$. For $\sigma_x > 0.50\cdot\sigma_0$ discrepancies are observable for $p_{\text{mean}} = 250$ N/mm². Apart from this, the comparison indicates a very high agreement between the experiments and the simulations.

The results of DP600 also highly agree. The comparison is shown in Fig. 21 to Fig. 30. In contrast to the DC04 results, the differences between the simulations and the experiments are negligible in the whole range from $\sigma_x = 0$ to $\sigma_x = \sigma_0$.

All in all, the results indicate the ability of the contact model to successfully determine the surface smoothing in dependency on the nominal normal load and the tensile bulk stress.

Fig. 11. $R_x$ of DC04-surface with $\sigma_x = 0$.

![Fig. 11](image1.png)

Fig. 12. $R_x$ of DC04-surface with $\sigma_x = 0.25\cdot\sigma_0$.

![Fig. 12](image2.png)

Fig. 13. $R_x$ of DC04-surface with $\sigma_x = 0.50\cdot\sigma_0$.

![Fig. 13](image3.png)
Fig. 14. $R_a$ of DC04-surface with $\sigma_x = 0.75 \cdot \sigma_0$.

Fig. 15. $R_a$ of DC04-surface with $\sigma_x = \sigma_0$.

Fig. 16. $R_q$ of DC04-surface with $\sigma_x = 0$.

Fig. 17. $R_q$ of DC04-surface with $\sigma_x = 0.25 \cdot \sigma_0$.

Fig. 18. $R_q$ of DC04-surface with $\sigma_x = 0.50 \cdot \sigma_0$.

Fig. 19. $R_q$ of DC04-surface with $\sigma_x = 0.75 \cdot \sigma_0$.

Fig. 20. $R_q$ of DC04-surface with $\sigma_x = \sigma_0$.

Fig. 21. $R_a$ of DP600-surface with $\sigma_x = 0 \cdot \sigma_0$.

Fig. 22. $R_a$ of DP600-surface with $\sigma_x = 0.25 \cdot \sigma_0$.

Fig. 23. $R_a$ of DP600-surface with $\sigma_x = 0.50 \cdot \sigma_0$.
4.3 Proposal Of A Modified Friction Law

In this section the CFL is extended based on the results of Sec. 0. The CFL describes the resulting friction shear stress with Eq. (1) in dependency on $\alpha$. The identification of $\alpha$ is easily derived with the half-space model and simpler than an experimental identification.

Fig. 31. Fraction of real contact area for DC04.

Fig. 31 shows the contact ratio for DC04 as well as a curve fit which is defined by:

$$\alpha = \sqrt{\tanh \left(b_1 \cdot \left( \frac{p_{\text{mean}}}{H_0} \right)^{c_1} \cdot \left( \frac{H}{H_0} \right)^{d_1} \cdot \left( 1 - \frac{\sigma_x}{H_0} \right)^{e_1} \right)^{\frac{1}{a_1}}}.$$  (23)
Therewith, $\alpha$ is not only a function of the normalized normal load $p_{\text{mean}}/H_0$ but also a function of the normalized current hardness $H/H_0$ and the normalized bulk stress $\sigma_x/H_0$. The parameters $a_1, b_1, c_1, d_1$ and $e_1$ are identified with the least squares method and shown in Table 3.

Table 3. Coefficients for $\alpha$ for initial contact.

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_1$</td>
</tr>
<tr>
<td>DC04</td>
<td>20</td>
</tr>
<tr>
<td>DP600</td>
<td>20</td>
</tr>
</tbody>
</table>

The contact ratio for DP600 and the corresponding curve fit are shown in Fig. 32.

As already mentioned, the friction model is able to consider un-/reloading. In order to retain this explicit feature, Eq. (3) is analogously to Eq. (2) enhanced to:

$$
\alpha = \sqrt{\tanh\left(\frac{b_2}{a(p_{h,H_0,H_x})}, \left(\frac{p_{\text{mean}}}{H_0}\right)^{c_2}, \left(\frac{H}{H_0}\right)^{d_2}\right)^{e_2}}
$$

(24)

where $a(p_{h,H_0,H_x})$ is the share between the real contact area and the apparent area. The latter occurs at the maximum contact pressure $p_h$ of former contacts. The hardness $H_0$ and the bulk stress $\sigma_x$ also describe the condition when the maximum contact pressure takes place. The parameters in Eq. (24) are also evaluated numerically and shown in Table 4. It must be noted that Eq. (24) does not depend on the normalized bulk stress, since the coefficient of $\sigma_x/H_0 \approx 0$ for both DC04 and DP600. A choice of four examples showing the conformity of Eq. (24) is presented in Fig. 33 to Fig. 36. Fig. 33 and Fig. 34 show $\alpha$ for DC04 with $\sigma_x = 0$ and 0.50-$\sigma_0$, respectively. Besides initial contact, un-/reloading with $p_h$ ranging from 100 N/mm$^2$ to 500 N/mm$^2$ in steps of 100 N/mm$^2$ is depicted. Fig. 35 and Fig. 36 show $\alpha$ for DP600 with $\sigma_x = 0.25\cdot\sigma_0$ and 0.75-$\sigma_0$, respectively. The maximum historic pressure ranges for DP600 from 100 N/mm$^2$ to 700 N/mm$^2$ in steps of 100 N/mm$^2$.

Table 4. Coefficients for $\alpha$ for un- & reloading.

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_2$</td>
</tr>
<tr>
<td>DC04</td>
<td>3.069</td>
</tr>
<tr>
<td>DP600</td>
<td>2.368</td>
</tr>
</tbody>
</table>

Fig. 32. Fraction of real contact area for DP600.

Fig. 33. $\alpha$ for DC04 and $\sigma_x = 0\cdot\sigma_y$.

Fig. 34. $\alpha$ for DC04 and $\sigma_x = 0.50\cdot\sigma_y$. 
5. CONCLUSION

The experimental study of this work investigates the influence of bulk stresses on the plastic surface smoothing of electric discharge textured sheet metal. The experimental setup is able to apply a tensile stress, which ranges between 0 and flow stress, to the blank. Simultaneously, the surface of the sheet metal is brought into contact with a stamp which is much harder than the sheet sample. The plastic smoothing of the sheet metal that is characterized by $R_a$ and $R_q$ indicates a dependency on the normal load and on the bulk stress. A half-space model which is enhanced to take into account work hardening is used to model the experimental outcome. The half-space model is able to successfully simulate the change of the surface roughness for both DC04 and DP600 with minor deviations. The half-space model can also identify the ratio between the real contact area and the apparent area. The ratio is a function of the normalized normal load, the normalized current hardness, and the normalized bulk stress. This fit can be combined with the CFL, as the CFL is the product of the shear strength and the contact ratio. This approach fulfills the demand to evaluate the resulting friction shear stress in dependency on the bulk stress.

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REFERENCES


