

A Method for Predicting Contact Strength and Life of Archimedes and Involute Worm Gears, Considering the Effect of Wear and Teeth Correction

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ABSTRACT

A new method developed by the author is used to investigate the effect of wear of worm wheel teeth on variations in their curvature radius and that of teeth correction in Archimedes and involute worm gears on their contact strength, wear and life. The effect of teeth correction on these variables is determined, for both constant and changed contact conditions. It has been found that when a positive correction coefficient is applied, the maximum contact pressures and wheel teeth wear decrease while the gear life increases; when the correction coefficient is negative, the trend is opposite. On changing contact conditions after wear limit of the teeth, the contact pressures are considerably lower, the gear life is practically the same, and the wear of worm wheel teeth is slightly higher than that in the case of constant conditions.

To accelerate calculations, the block-cumulative method developed by the author and not the specified step-by-cumulative method are used. The computational time is reduced proportionately to the size of the block of the number of interactions.

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1. INTRODUCTION

Worm gears are widely used in machine design because they ensure considerable changes in torque and revolutions frequency with small dimensions. When gear elements are in mesh, sliding friction occurs, causing wear of the worm wheel teeth. It is vital to be able to predict gear life and wear of worm wheel teeth already the stage of gear design. Despite the obvious need for that, the methods for predicting abrasive wear of worm gear teeth given in the literature

[1-3] cannot be applied to uncorrected and corrected gears, especially at boundary lubrication. Based on elastohydrodynamic lubrication theories, the studies [1,2] report the results of prediction of teeth wear in an uncorrected gear by a modified Archard law, using a model allowing for variations in contact pressures and oil film thickness in contact area. However, it should be noted that the Archards law of abrasive wear used in the above-mentioned works, assuming linear dependence on slip velocity and contact pressures, in the

case of elastohydrodynamic and also boundary lubrication does not work.

Based on the author generalized method for prediction of wear at sliding friction (dry, mixed, boundary) [4], where the mechanism of fatigue wear is assumed, which has been confirmed by the works of the author as well as in literature by other researchers, the author developed a method [5] enabling prediction of contact parameters, worm wheel teeth wear and gear life. This paper reports the results obtained with the proposed method with respect to determination contact and tribological variables describing Archimedes and involute worm gears, considering the effect of teeth wear and correction on these parameters.

2. MATHEMATICAL MODEL OF WEAR AT SLIDING FRICTION

In worm gears, at oil lubrication, sliding friction occurs between the worm coils and the worm wheel teeth at torque transfer. Assuming that the wear rate of the elements of the tribosystem (see - the left side of equation (1)) is functionally related to the level of specific friction force (right side of the equation), the kinetics of wear in a sliding tribological system is described by a system of linear integer equations [4]:

$$\frac{1}{v_j} \frac{dh_{kj}}{dt} = \Phi_k^{-1}(\tau), \quad k=1;2. \quad (1)$$

Experimental values of the function are approximated by the following formula:

$$\Phi_k(\tau) = C_k (\tau_{sk} / \tau)^{m_k} \quad (2)$$

where $\tau_{sk} \approx 0.35R_{mk}$.

The experimental values of functions are determined as follows [4]:

$$\Phi_i(\tau)_i = L / h_i, \quad (3)$$

where $L = vt$.

The unit friction force is described by the Amontons-Coulomb law of friction:

$$\tau_j = fp_j \quad (4)$$

The maximum contact pressured $p_j = p_{j\max}$ is determined with the Hertz formula, depending

on the number of meshing pairs w of the worm wheel teeth:

$$p_{j\max}^{(w)} = 0.564 \sqrt{N' / w \theta \rho_j b} \quad (5)$$

where $\theta = (1 - \mu_1^2) / E_1 + (1 - \mu_2^2) / E_2$.

2.1 Linear wear of the worm wheel teeth ($p_{j\max} = \text{const}$)

By integration of equation (1), considering function (2) and formulas (4), (5) and assuming that the contact pressures const, we get a formula for calculating unit linear wear of gear teeth during a single time of tribocontact interaction:

$$h'_{2j} = \frac{v_j t'_j \left(fp_{j\max}^{(w)} \right)^{m_2}}{C_2 (\tau_{s2})^{m_2}} \quad (6)$$

where $t'_j = 2b_j^{(w)} / v_j$.

According to the Hertz formula:

$$2b_j^{(w)} = 2.256 \sqrt{\Theta N' \rho_j / bw}$$

The sliding rate is:

$$v_j = \sqrt{(v'_j)^2 + (v''_j)^2} \quad (7)$$

Accordingly,

$$v'_j = \frac{\omega_1 x}{\cos \gamma_A} \quad (8)$$

where $tg \gamma_A = mz_1 / 2x$, $\omega_1 = \pi n_1 / 30$.

$$v''_j = e_{pAj} \omega_2, \quad \omega_2 = \omega_1 / u \quad (9)$$

It should be noted that the dominant effect on the resulting slip speed is velocity v' .

The meshing force N' is determined by the formula:

$$N' = \frac{2T}{d_1 \cos \alpha_{pxj} \sin(\gamma + \rho')} \quad (10)$$

where:

$$T = 9550 \cdot 10^3 (N / n_1) \text{ and}$$

$$\rho' = \arctg(f / \cos \alpha).$$

The Archimedes worm about the trapezoidal profile (Fig. 1) is:

$$\rho_j = \left(\frac{d_2}{2} \sin \alpha_{xj} + e_{pAj}\right) \quad (11)$$

the involute worm is:

$$\rho_j = \frac{\rho_{1j} \rho_{2j}}{\rho_{1j} + \rho_{2j}} \quad (12)$$

The curvature radius of involute worm thread profiles ρ_{1j} and worm wheel teeth ρ_{2j} .

$$\rho_{1j} = -\frac{r_b \operatorname{tg} \alpha_{cj}}{\cos^3 \alpha_{pxj} \operatorname{tg} \gamma_b \cos^2 (\alpha_{cj} + \varepsilon_j)}$$

$$\rho_{2j} = \frac{\rho_{1j} r_2 \sin \alpha_{pxj} + \rho_{1j} e_{pAj} - e_{pAj}^2}{r_2 \sin \alpha_{pxj} + \rho_{1j} - e_{pAj}} \quad (13)$$

The worm wheel teeth have involute profiles (Fig. 1).

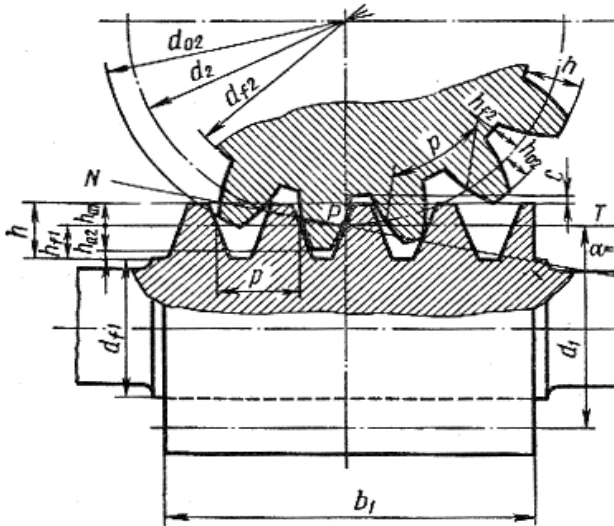


Fig. 1. Schematic design of a worm gear.

The coordinate x is in the range of $x_A < x < x_B$. Accordingly $x_A = r_{f1} + 0.2m$ and $x_B = r_{a1}$.

The section of meshing $[x_A, x_B]$ must be proportionately divided into smaller sections: $x_A = j_A = j_1, x_2 = j_2, x_3 = j_3, \dots$ and $x_B = j_n = j_B$.

3. GEAR GEOMETRY

The geometry of the analysed worm gears (Fig. 1) is described by the following relationships:

$$r_{f1} = 0.5(d_1 - 2h_{f1}), h_{f1} = 1.2m$$

(where $\gamma \leq 15^\circ$), $h_{f1} = 1.2m_n$ (where $\gamma > 15^\circ$);

$$\tan \gamma = mz_1 / d_1, \quad d_1 = qm;$$

$$r_{a1} = 0.5(d_1 + 2h_{a1}), h_{a1} = m$$

(where $\gamma \leq 15^\circ$), $h_{a1} = m_n$ (where $\gamma > 15^\circ$);

$$r_2 = 0.5z_2m, r_2 = 0.5d_2,$$

$$z_2 = uz_1, q = 2(1 + \sqrt{z_2});$$

$$e_{pAj} = \frac{r_1 - x}{\sin \alpha_{pxj}}, r_1 = 0.5d_1, b = 2m\sqrt{q+1}.$$

Other geometrical parameters:

- Archimedes worm gear:

$$\alpha_{pxj} = \arctan(-\tan \alpha_{xj}), \tan \alpha_{xj} = \frac{180}{\pi} \frac{mz_1}{2x},$$

- Involute worm gear:

$$r_b = 0.5d_1 \cos \alpha_c, \tan \alpha_c = \tan \alpha_n / \sin \gamma,$$

$$\alpha_{cj} = \arctan \frac{\sqrt{x^2 - r_b^2}}{r_b},$$

$$\alpha_{pxj} = \arctan \left(-\tan \gamma_b \frac{\sqrt{x^2 - r_b^2}}{x} \right),$$

$$\tan \gamma_b = \frac{mz_1}{d_1 \cos \alpha_c}, \varepsilon_j = \frac{180}{\pi} \frac{\sqrt{x^2 - r_b^2}}{r_b}.$$

4. LINEAR WEAR OF TEETH AND LIFE OF THE WORM GEAR ($p_{j\max} = \text{const}$)

The wear of the worm wheel teeth within one hour of gear operation is calculated in the following way:

$$\bar{h}_{2j} = 60n_2h'_{2j}, n_2 = n_1 / u \quad (14)$$

The life of the worm gear for the acceptable wear h_{2*} of the worm wheel teeth is calculated according to the formula:

$$t_* = \left(h_{2*} / \bar{h}_{2j} \right) \quad (15)$$

5. LINEAR WEAR OF THE WORM WHEEL TEETH ($p_{j\max} = \text{var}$)

In this case, the formula (6) has the form:

$$h'_{2jn} = \frac{v_j t'_{jh} \left(f p_{jh \max}^{(w)} \right)^{m_2}}{C_2 (\tau_{s2})^{m_2}} \quad (16)$$

As a result, the radius of curvature of the teeth increases with every revolution of the worm wheel, and the maximum tribocontact pressures decreases with increasing the width of the contact area. Accordingly,

$$\begin{aligned} p_{jh \max} &= 0.564 \sqrt{N' / bw \theta \rho_{jh}}, \\ 2b_{jh} &= 2.256 \sqrt{\theta N' \rho_{jh} / bw} \end{aligned} \quad (17)$$

where $\rho_{jh} = \rho_{2jh}$ - Archimedes worm gear,

$$\rho_{jh} = \frac{\rho_{1j} \rho_{2jh}}{\rho_{1j} + \rho_{2jh}} \text{ - involute worm gear,}$$

$$t'_{jh} = 2b_{jh} / v_j.$$

5.1 Definition variations in teeth curvature radius due to wear

The proposal of a method is to determine the effect of linear wear of gear teeth on the curvature radius. The variable curvature radii ρ_{2jh} of teeth profiles at j -th point of contact are expressed:

$$\rho_{2jh} = \rho_{2j} + \lambda_h \sum_1^{n_{2*}} h'_{2jn} \quad (18)$$

Therefore, for numerical resolution, a step-by-step calculation of the following parameters is performed: h'_{2jn} , ρ_{2jh} , ρ_{jh} , $p_{jh \max}$, $2b_{jh}$, t'_{jh} .

To accelerate the calculations, it is expedient to use the block-cumulative method developed by the author and not the specified step-by-cumulative method. This method is based on an assumption that, during a given number of wheel revolutions n_2 (interaction block B), the parameters h'_{2j} , $p_{j \max}$, $2b_j$, t'_j are constant. A subsequent block allows for a change in these parameters due to wear, and so the calculations are made for new values. The computational time is reduced proportionately to the size of the block of the number of interactions. For example, in the given solution (item 8) when the block size is $B = 10(60n_1 / u) = 33177$ revolutions, then this time will be 33177 times smaller than in the case of the step-by-cumulative method.

Hence, according to the block-cumulative method variable curvature radii:

$$\rho_{2jh} = \rho_{2j} + \lambda_h \sum_1^B h'_{2jn} \quad (19)$$

6. TOTAL WEAR AND LIFE OF THE WORM GEAR ($p_{j \max} = \text{var}$)

The total wear of worm wheel teeth determined by the above step-by-cumulative and block-cumulative methods is:

$$h_{2jn} = \sum_1^{n_{2*}} h'_{2jn}, h_{2jB} = \sum_1^{n_{2*}} h_{2jB}, \quad (20)$$

where $h_{2jB} = \sum h'_{2j}$.

The gear life after revolutions n_{2*} of a worm wheel when acceptable wear is reached:

$$t_* = n_{2*} / 60n_2. \quad (21)$$

7. WORM WHEEL TEETH CORRECTION

In worm gears, we can only apply correction to the worm wheel teeth.

The interaxial distance is:

$$a_k = a + x_2 m \quad (22)$$

where $a = (d_1 + d_2) / 2$, $x_2 \leq \pm 1$.

The reference diameter of the worm in an uncorrected gear:

$$d_{w1} = d_1 + x_2 m \quad (23)$$

Therefore, the distance e_{pAj} between the j -th point of contact and the point of contact will be:

$$e_{pAj} = \frac{r_{w1} - x_j}{\sin \alpha_{pxj}}.$$

Other geometrical parameters are determined in accordance with the formulas for uncorrected worm gears.

8. NUMERICAL SOLUTION

The computations were performed using the following set of data: $N = 3.5$ kW, $n_1 = 1410$ rpm, $m = 6$ mm, $z_1 = 2$, $u = 25.5$, $f = 0.05$, $q = 8$; worm -

hardened steel grade 45 (HRC 50) described by $E_1 = 2.1 \cdot 10^5$ MPa, $\mu_1 = 0.3$; worm wheel ring – bronze CuSn6Zn6Pb6 described by $E_2 = 1.1 \cdot 10^5$ MPa, $\mu_2 = 0.34$; $C_2 = 7.6 \cdot 10^6$, $m_2 = 0.88$; $\tau_{s2} = 75$ MPa; for $j = 1; 2; 3; 4$ and 5 , respectively $x = 18; 20; 22; 24$ and 26 mm; $h_{2*} = 0.5$ mm; $\lambda_h = 100$; $B = 10(60n_1/u) = 33177$ revolutions (10 hours of work); with double-pair meshing.

The numerical results are given in Figs. 2-7: a) Archimedes worm gear, b) involute worm gear. Solid lines denote constant conditions of contact, broken lines mark variations in the reduced curvature radius due to wear.

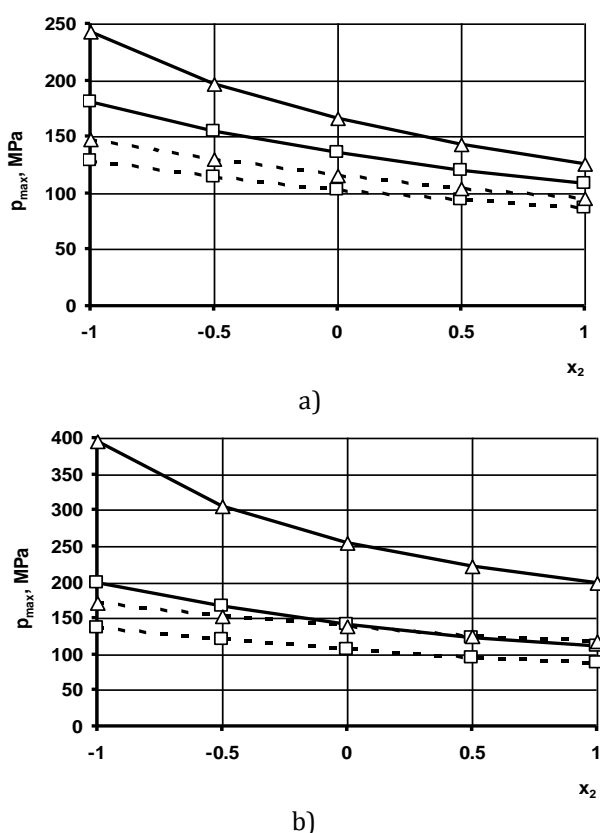


Fig. 2. Wear and correction versus p_{max} : $\square - p_{max}$ when $j = 1$; $\triangle - p_{max}$ when $j = 5$.

Figure 2 shows the relationships between the maximum contact pressures p_{max} and the displacement coefficient x_2 on entering the mesh ($j = 1$) and on leaving the mesh ($j = 5$). It was found that the positive correction of the worm wheel teeth leads to reduced pressures, while the negative correction results in their increase compared to the observations made regarding the uncorrected gear. This results in an increase in the teeth curvature summary

radius $\rho_z = \rho$ at their acceptable wear $h_{2*} = 0.5$ mm when x_2 is positive and it decrease at negative values (Fig. 5).

As a result of the applied teeth correction, when $x_2 > 0$, ρ and ρ_z increase and the gear life t_{min} increases too (Fig. 3). When $x_2 < 0$, ρ and ρ_z decrease, which leads to reduced gear life. Taking into account the teeth wear, the gear life will be slightly shorter than that in the case with constant conditions of interaction.

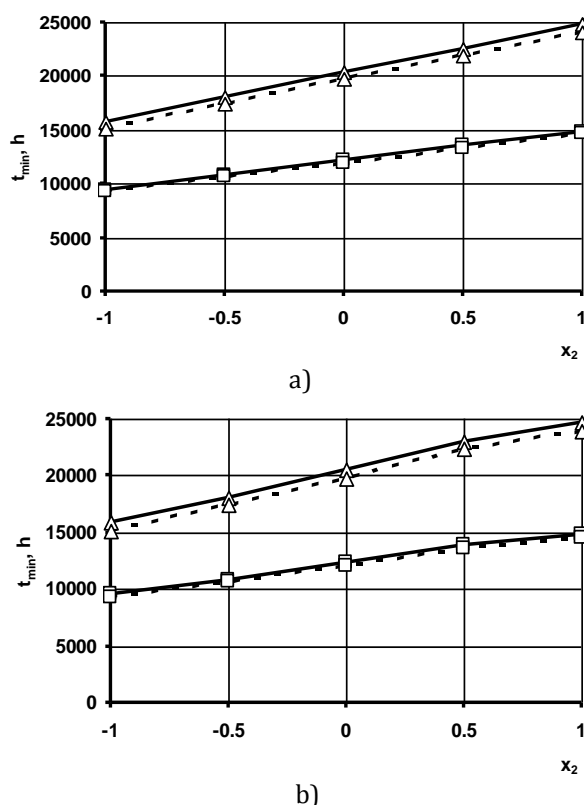


Fig. 3. Wear and correction versus gear life t_{min} : $\triangle - t_{min}$ when $h_{2*} = 0.3$ mm; $\square - t_{min}$ when $h_{2*} = 0.5$ mm.

The linear wear of gear teeth during one hour of operation, with contact conditions maintained constant (friction path $2b_j$), will be lower than under variable contact conditions (friction path $2b_{jh}$) due to wear (Fig. 4). When positive correction coefficients x_2 are applied, the wear \bar{h}_2 is lower (Fig. 4) than that of the uncorrected gear.

As a result of teeth wear, the curvature radii ρ_2 of the wheel teeth (Archimedes worm) and the reduced curvature radii $\rho_z = \rho$ (involute worm) significantly increase, when compared to their initial values (Fig. 5).

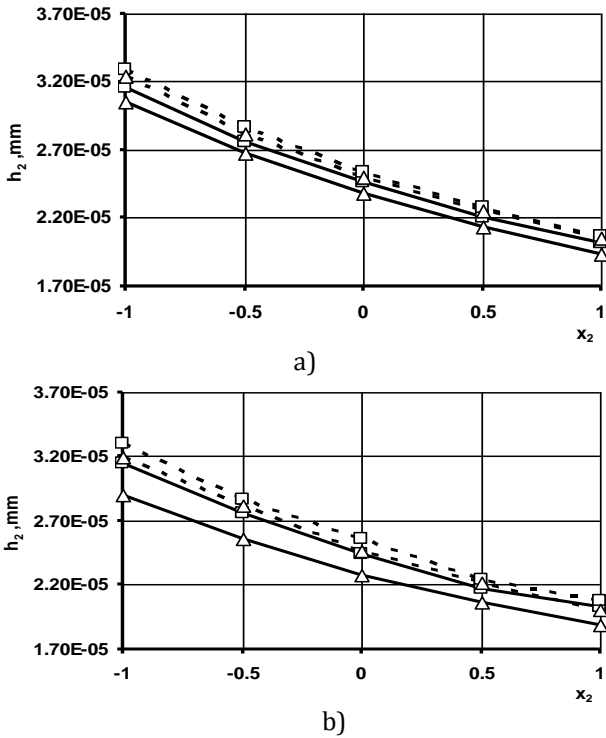


Fig. 4. Wheel teeth wear per hour versus change in x_2 : \square - $\bar{h}_{2\max}$ when $j=1$; \triangle - $\bar{h}_{2\min}$ when $j=5$.

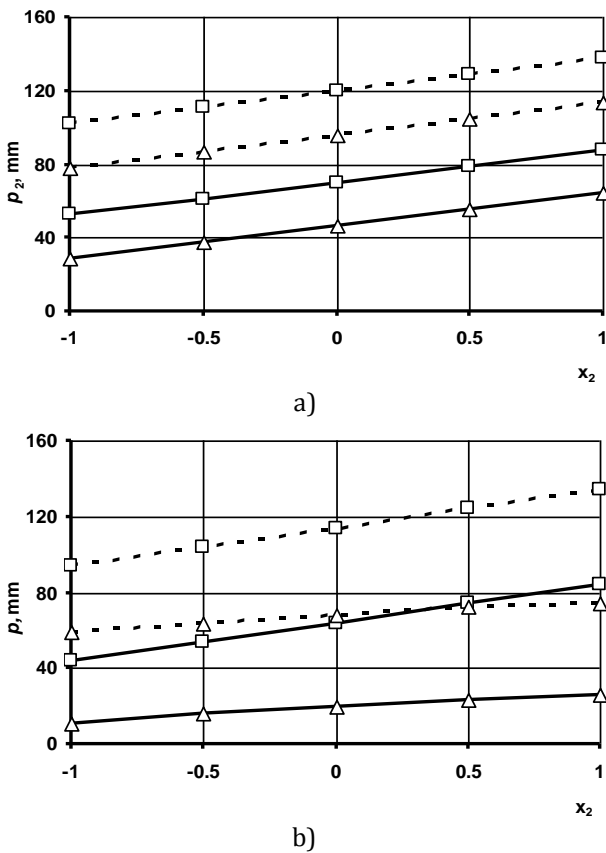


Fig. 5. Wear and correction versus curvature radii change: \square - $\rho_{2\max}$ (ρ_{\max}) when $j=1$; \triangle - $\rho_{2\min}$ (ρ_{\min}) when $j=5$.

In both gear types, the sliding rate does not change at j -th contact points, in either constant or variable conditions of interaction (Fig. 6).

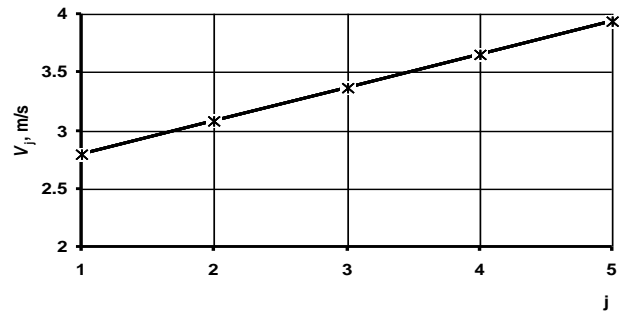


Fig. 6. Variations in sliding rate over tooth height.

The effect of gear teeth correction on the contact area width $2b_j$ is insignificant when $p_{\max} \cdot 2b_j = \text{const}$; the value of $2b_{jh}$ is affected more significantly when $p_{\max} \cdot 2b_j = \text{var}$ due to wear of the worm wheel teeth (Fig. 7). The contact area width considerably increases due to wear of the gear teeth.

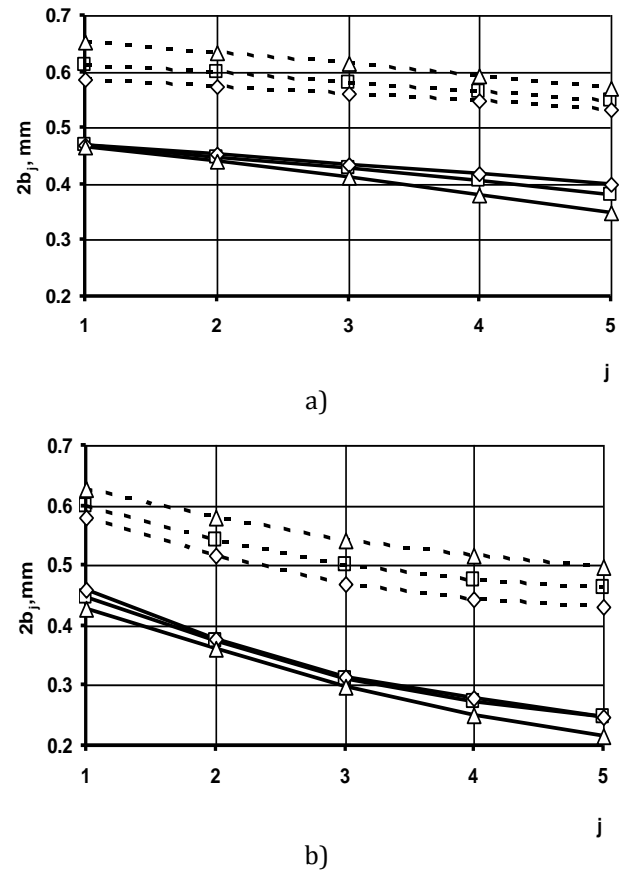


Fig. 7. Wear and correction versus contact area width during one interaction between tooth and worm coil: \triangle - $x_2 = -1$; \square - $x_2 = 0$; \circ - $x_2 = 1$.

The proposed method can be used to establish qualitative and quantitative relationships with respect to the effect of teeth correction and wear on the load capacity, life, wear, sliding velocity and contact area width of the worm gear.

9. CONCLUSION

1. It has been found that, in contrast to uncorrected gears, the maximum contact pressures decrease when the correction coefficient x_2 is positive. When this coefficient is negative – the pressures increase (Fig. 2).
2. The life of the worm gear increases at $x_2 > 0$ but decreases at $x_2 < 0$ (Fig. 3).
3. The wear of the worm wheel teeth decreases at $x_2 > 0$ and increases at $x_2 < 0$, in contrast to the case when $x_2 = 0$ (Fig. 4).
4. The tooth curvature radii increase with increasing x_2 and they decrease with decreasing x_2 (Fig. 5).
5. Gear correction does not have a significant impact on the contact area width.
6. Negative correction leads to decreased load capacity and life of the worm gear and increased wear of worm wheel teeth, hence it is not recommended.

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NOMENCLATURE

- a is the interaxial distance in an uncorrected gear,
 b_2 is the worm wheel width,
 C_k, m_k are the indicators of wear resistance of tribological pair materials in select wear conditions,
 d_1 is the reference diameter of the worm,
 d_2 is the reference diameter of the worm wheel,
 e_{pA} is the distance of j point from the contact point,
 f is the sliding friction coefficient,
 q is the diametric quotient of the worm gear,
 h_k is the linear wear of elements of a tribological pair,
 h_i consumption of samples of tribological vapor materials,
 h_{f1} is the height of worm thread base,
 h_{a1} is the height of head of the worm coil,
 h'_{2jn} is the linear wear of teeth during a single interaction, decreased due to changes in ρ_{2jh} , t'_{jh} , P_{jhmax} ,
 h_{2jB} is the wear of teeth during a block of interaction,
 h_{2*} is acceptable wear the teeth of the worm wheel,
 i load level,
 j point of contact between elements of a kinematic pair (worm – worm wheel tooth),
 k denotes the number of an element of the tribological pair (1 – worm, 2 – worm wheel),
 L the road of friction,
 m is the axial modulus of meshing,
 $m_n = m \cos \gamma$ is the normal modulus of meshing,
 n_2 is the number of revolutions of the worm wheel per minute,
 n_{2*} is the number of revolutions of the worm wheel when the acceptable worm wheel teeth wear h_{2*} is reached,

- N' is the meshing force,
- N is the transmitted power,
- n_1 is the number of revolutions of the worm,
- $p_j = p_{j\max}$ are the maximum contact pressured determined with the Hertz formula, depending on the number of meshing pairs w of the worm wheel teeth,
- $p_{jh\max}$ are the maximum current tribocontact pressures.
- R_m is the temporary tensile strength of materials,
- r_{f1} is the radius of a circle of worm cavity,
- r_{a1} is the radius of a circle of worm coil prongs,
- r_b is the radius of a basic circle of worm coils,
- t is a period of wear,
- t'_j is the time of contact between the meshing elements at j -th point on the friction path with a length equal to the contact area width $2b_j^{(w)}$,
- T is the torque transmitted by the worm,
- u is the gear ratio,
- v is the sliding rate at j -th point of contact between elements of a kinematic pair,
- v'_j is a sliding rate during worm gear revolution,
- v''_j is a sliding rate at the point of contact between the worm wheel tooth and worm coil,
- w is the number of meshing pairs between worm coils and worm wheel teeth,
- x_2 is the correction coefficient,
- z_1 is the number of worm coils,
- z_2 is the number of teeth in the worm wheel,
- α_n is the angle of meshing,
- α_c is a face pressure angle,
- α_{cj} is the face pressure angle at j -th point,
- γ is the angle of elevation of the screw line of worm coils,
- γ_b is the angle of inclination of flank pitch line on reference diameter of the gear,
- ε is an angular coordinate in every pitch (degrees),
- $\Phi(\tau)$ is the characteristic function of wear resistance of tribological pair materials under select conditions,
- λ_h is a non-dimensional coefficient of wear impact,
- μ_k, E_k are Poisson's ratio and Young's modulus of material of the worm wheel, respectivity,
- ρ_j is the reduced curvature radius between worm coils and worm wheel teeth at j -th point of meshing,
- ρ_z is the reduced radius of curvature of the worm gear,
- ρ is the reduced radius of curvature of the involute worm gear;
- ρ_{jh} is the reduced radius of curvature of the involute worm gear a result of the wear tooth; wear of the steel worm is omitted,
- $\rho_{2,jh}$ is the radius of curvature of the Archimedes worm gear,
- ρ' is a friction angle,
- τ is the unit friction force affecting wear rate of materials,
- τ_{sk} is the temporary shear strength of gear material,
- θ is the Kirchhoff modulus,
- ω_1 is an angular velocity of the worm.