

Friction Forces at the Wheel's Contact with the Ground in a Turning Vehicle

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ABSTRACT

Tangential ground reactions are friction forces in their essence. Problems in the description of the frictional contact of the wheel with the ground arise during the curvilinear motion of a vehicle. There is a force and friction torque during rotational relative slip. The single physical essence of the friction force and the moment of friction determine their mutual interconnection. The authors of the article have developed a new mathematical model of the formation of forces in contact with the ground when a vehicle makes a turn. This model is based on the theory of friction for plane-parallel relative slip. The friction force and the frictional moment are integral functions of the coordinates of the instantaneous center of slip. Changing the limits of integration allowed us to take into account the shape and size of the contact. The introduction of normal pressure into the integral made it possible to take into account any law of its change. The use of different friction coefficients in the longitudinal and transverse directions made it possible to take into account the anisotropic properties of the contact. Variable friction coefficients allowed us to take into account the elastic properties of the wheel and the ground.

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1. INTRODUCTION

The dynamics of vehicle motion is currently of great interest among the scholars [1-4]. Any suggested mathematical model of motion contains reactions from the ground, where tangential forces at the wheel's contact with the ground serve as friction forces in their essence [5-6].

Different assumptions describing ground reaction subsequently after the contact define the application sphere of the motion model itself [7-8]. However, authors do not always disclose the assumptions used, which sometimes results in the use of motion models in inadmissible areas and, consequently, leads to gross calculation errors. One of the reasons for this situation is to use empirical dependencies instead of the laws

of mechanics [9]. The purpose of our research was to analyze the most common errors when describing the tangential friction forces that occur in the contact of the wheel with the ground when the vehicle makes a turn.

2. PROBLEM ANALYSIS

When a wheel is in contact with the ground different elementary friction forces arise during motion. The problem of describing the force interaction is reduced to replacing the sum of all elementary friction forces with one resultant friction force. Many authors have assumed that the total friction force acts in the centre of the contact zone [10-11].

This assumption is incorrect. The laws of mechanics show that the body can move only directly, without turning under the influence of a force applied in the centre of pressure. Consequently, one resultant force in the centre of the pressure of the contact area promotes direct sliding. This is contrary to the nature of the turn. Changing the direction of the resultant force only changes the sliding trajectory, but does not ensure the rotation of the contact area, which is characteristic of motion along a curve. Consequently, in a general case of curvilinear motion, the resultant force is displaced relative to the contact centre, due to which the wheel slides along plane-parallel laws of motion.

The moment of friction appeared when the total friction force was transferred to the center of contact. The result of bringing all elementary friction forces to the center of the contact zone is the resultant force P and the resultant (stabilizing) moment M [12].

Different scholars use different empirical dependencies to calculate stabilizing moment M [12-14]. Despite the variety of formulas used to calculate moment M , the force interaction models are united by the fact that the value of the resulting force received in all the studies is equal to the friction limit $P = \varphi_{max} N$.

This assumption is erroneous too. Resulting force P_p results from the sum of elementary forces $P_p = \sum dP_{pi}$ identical by value and direction (Fig. 1a). If it reaches the friction limit, each elementary force dP_{pi} in a point also

reaches the friction limit. Moment M is the result of the sum of other elementary forces dP_{mi} with different modules and directions (Fig. 1b). The resulting force at each point is determined by the vector sum of both elementary forces $d\vec{P}_i = d\vec{P}_{pi} + d\vec{P}_{mi}$ (Fig. 1c).

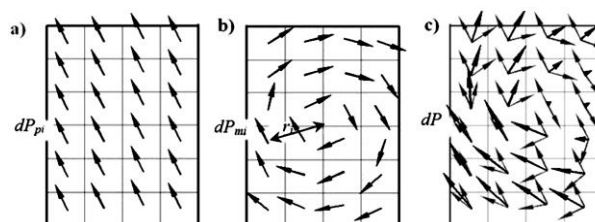


Fig. 1. Formation of the force factors in a wheel-ground contact when a vehicle makes a turn.

Coulomb's law must be satisfied at each point $dP_i \leq \varphi_{max} dN$. In other words, one component reaches the limit of friction only in the absence of the other. If $dP_{pi} = \varphi_{max} dN$, then $dP_{mi} = 0$, and vice versa.

In other words, during plane-parallel sliding, the values of the resultant friction force P and the moment of friction M are interconnected (the greater the friction torque M is, the smaller the resultant friction force P is). This is explained by the fact that the force P and the moment M have the same physical nature – friction limited by the limit of friction in the aggregate. Therefore, when a vehicle makes a turn, the maximum value of the total friction force is always less than the friction limit $P < \varphi_{max} N$. This proves that the principle of superposition is unacceptable in the case of friction. Any empirical dependencies are limited by the law of similarity and, therefore, are not widely used.

The purpose of this study was to develop a model of force interaction of the wheel with the ground at the turn of the vehicle based on the laws of mechanics and the theory of friction, which allows us to take into account the motion mode of a wheel, its slip, shape and size of the contact zone pattern and interaction anisotropy.

3. GENERAL MODEL OF FRICTION FORCES WHEN THE WHEEL CONTACTS THE GROUND

The first studies on friction forces during plane-parallel motion were completed in 1892 by N.N.

Shiller. The building process of the base force interaction model is described in detail in [15]. Force factors P_x, P_y, M in a wheel-ground contact in (1) are presented as functions of the coordinates x, y of instant slip centre (point C in Fig. 2).

$$\left. \begin{aligned} P_x &= \varphi q \iint_{\xi\eta} \frac{y-\eta}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \\ P_y &= -\varphi q \iint_{\xi\eta} \frac{x-\xi}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \\ M &= \varphi q \iint_{\xi\eta} \sqrt{(x-\xi)^2 + (y-\eta)^2} d\xi d\eta \end{aligned} \right\}, \quad (1)$$

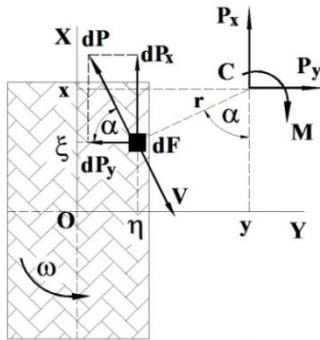


Fig. 2. Formation of force factors in a wheel-ground contact.

4. ADDITIONAL MODEL OPTIONS

4.1. The shape and size of a wheel-ground contact

The model takes into account the shape and size of the wheel contact with the ground by changing the limits of integration. Current coordinates ξ, η change:

1. rectangular contact form has limits of integration $-0.5b \leq \eta \leq 0.5b$ $-0.5L \leq \xi \leq 0.5L$,

2. the oval shape has limits of integration

$$-c \leq \eta \leq c, \quad -\frac{d}{c} \sqrt{c^2 - \xi^2} \leq \xi \leq \frac{d}{c} \sqrt{c^2 - \xi^2},$$

Contact area F was calculated for the rectangle $F = bL$ and for the oval $F = \pi cd$.

4.2. The law of normal pressure variation in a wheel-ground contact

The average value of normal pressure $q = N/F$ is often used in calculations for wheeled vehicles. For a more accurate solution, it is necessary to consider the actual law of pressure variation on the contact area of the wheel with the ground. In this model, the real distribution of

normal pressure was considered by introducing q in formulas (1) under the integral sign.

1. pressure distribution along rectangular

contact (Fig. 3a) $q(\xi, \eta) = q_{max} \sqrt{1 - (\eta/b)^2}$,

2. pressure distribution along oval contact (Fig. 3b)

$$q(\xi, \eta) = q_{max} \sqrt{1 - (2\eta/c)^2 - (2\xi/d)^2}.$$

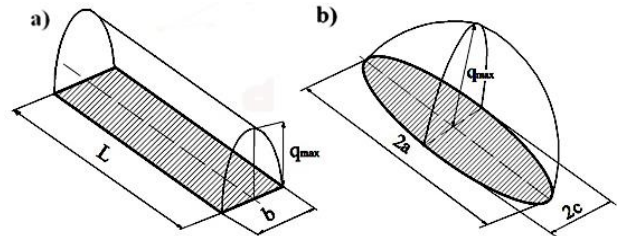


Fig. 3. Real laws of pressure distribution in (a) rectangular contact, (b) oval contact.

4.3. The anisotropic properties of a wheel-ground contact

The presence of irregular tread relief leads to different friction properties in the wheel-ground contact in the lateral and transverse direction. Problems of anisotropic friction are described in detail in article [16]. Axial force P_x was expressed by the axial friction coefficient φ_{xmax} and lateral force P_y by the lateral friction coefficient φ_{ymax} . The every elemental moment of friction M was calculated as the sum of the moment from the longitudinal and transverse component $dM = ydP_x + xdP_y$.

The integral dependencies of the force friction factors (1), considering different friction coefficients through the length of φ_{xmax} and breadth of φ_{ymax} of the wheel rolling plane, looked as follows:

$$\left. \begin{aligned} P_x &= \iint_{\xi\eta} \frac{q\varphi_{xmax}(y-\eta)}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \\ P_y &= -\iint_{\xi\eta} \frac{q\varphi_{ymax}(x-\xi)}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \\ M &= \iint_{\xi\eta} q \frac{\varphi_{xmax}(y-\eta)^2 + \varphi_{ymax}(x-\xi)^2}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \end{aligned} \right\} \quad (4)$$

4.4. The elastic properties of the wheel and ground

We used variable friction coefficient φ depending on slip value S to consider the elastic

behavior of the pneumatic tire and ground deformation. In this case, coefficient φ was used as an elementary specific tangential force $\varphi = dP/dN$. The nature of the change in specific tractive force φ from skid S is shown in Fig. 4.

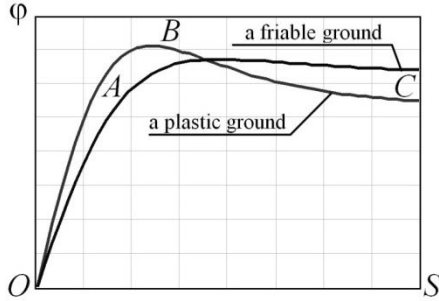


Fig. 4. Experimental dependence of specific tangential force on slip.

This curve describes all interaction stages:

1. linear section OA characterizing elastic deformation of the tire and the ground (pseudo-slip),
2. section AB characterizing the breakaway of the ground surface layer only typical of plastic soils,
3. section BC characterizing the full wheel slip with regard to the ground.

There are many different functions describing this dependence of $\varphi(S)$ today [17-19]. The best formula has the smallest number of empirical coefficients. Formula (5) had an advantage, because it contains only two empirical coefficients a_1 and a_2 [20-21].

$$\varphi = \varphi_m \left(1 + \frac{a_1}{ch(dS/a_2)} \right) th \left(\frac{dS}{a_2} \right) \quad (5)$$

The advantage of formula (5) is the ability to describe any type of soil:

1. at $a_1 = 0$ we receive loose soil (curve $OABC$),
2. at $a_1 \neq 0$ we receive plastic soil (curve OBC).

The dependence of the lateral force P_y on lateral slip S_y looks analogous. Anisotropic properties were reflected in different empirical coefficients a_1 and a_2 .

The introduction of the function (5) in the sub-integral dependencies (4) allowed us to consider the lateral and transverse elastic properties of the pneumatic tire and ground deformation in both directions. They have the following form:

$$\left. \begin{aligned} P_x &= \iint_{\eta\xi} \frac{q\varphi_{x\max} \left(1 + \frac{a_1}{ch(dS_x/a_2)} \right) th \left(\frac{dS_x}{a_2} \right) (y-\eta)}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \\ P_y &= - \iint_{\eta\xi} \frac{q\varphi_{y\max} \left(1 + \frac{a_1}{ch(dS_y/a_2)} \right) th \left(\frac{dS_y}{a_2} \right) (x-\xi)}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \\ M &= \iint_{\eta\xi} q \frac{\varphi_{x\max} \left(1 + \frac{a_1}{ch(dS_x/a_2)} \right) th \left(\frac{dS_x}{a_2} \right) (y-\eta)^2}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta + \\ &+ \iint_{\eta\xi} q \frac{\varphi_{y\max} \left(1 + \frac{a_1}{ch(dS_y/a_2)} \right) th \left(\frac{dS_y}{a_2} \right) (x-\xi)^2}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \end{aligned} \right\} \quad (6)$$

The influence of tire and ground elasticity on the interdependence of force $P(x, y)$ and moment $M(x, y)$ is shown in Fig. 5.

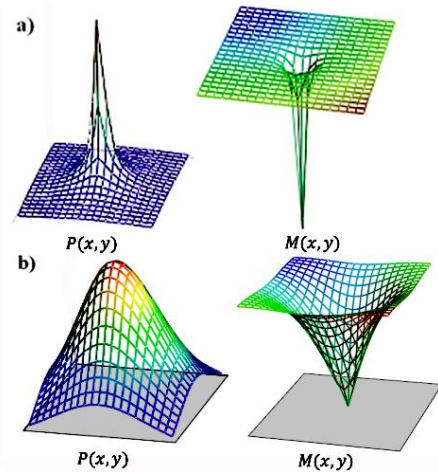


Fig. 5. Dependence of friction force P and moment M on coordinates x, y of the instant slip centre in space (a) without taking into account elastic properties in contact and (b) taking tire and ground elastic properties into account.

5. CONCLUSION

The model of force interaction in contact describes the force factors P_x, P_y, M as functions of coordinates x, y of instant slip centers. This approach has several strong points:

1. the basis of the suggested approach is formed by the laws of friction mechanics but not by empirical dependencies, which considerably extended its sphere of application,

2. force factors are functions of the coordinates of the instantaneous center of slip. This translated the force problem to the kinematics of plane-parallel motion and allowed us to solve power and kinematic problems together,
3. describing the force factors for each wheel individually allowed for the consideration of various wheel motion modes (driven, driving, brake),
4. the introduction of different adherence coefficients through the length and breadth of the wheel rolling plane allowed for the consideration of the interaction anisotropy in view of the tread pattern,
5. integral dependencies allowed for the consideration of the contact area pattern shape (oval or rectangle) by changing the integration limits,
6. the introduction of normal pressure q under the integral mark allowed for the consideration of the real law of the vertical load distribution along the contact area.

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NOMENCLATURE

M friction torque in the wheel-ground contact,

\mathbf{P} total force of friction in the contact of the wheel with the ground,

P_x longitudinal component of friction force in the wheel-ground contact,

P_y transverse component of friction force in the wheel-ground contact,

x, y coordinates of instant slip center,

ξ, η current coordinates of the elementary point of contact,

φ current value of friction coefficient,

φ_m friction coefficient corresponding to maximum slip,

φ_{max} maximum coefficient of friction,

$\varphi_{xmax}, \varphi_{ymax}$ maximum coefficient of friction in the longitudinal and transverse direction,

dS elementary slip in the contact point with coordinates ξ, η ,

dS_x elementary slip in the longitudinal direction,

dS_y elementary slip in the transverse direction,

N normal force in the wheel-ground contact,

q normal contact pressure,

F contact area of the wheel-ground,

L, b length and width of a rectangular contact,

c, d semi-axes of the oval contact.