

Analysis of the Energy Balance of Friction on the Rolling Contact

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ABSTRACT

The state of elastic rolling of the wheel is considered as a special manifestation of the highly developed evolution of the plastic contact of rolling friction. It is shown that for the material point of contact of smooth rolling surfaces, their initial elasticity forms a pair of elastic roughnesses – the sliding friction model. The Carter's diagram is analyzed and its interpretation is given as an analog of the metal tension curve. This diagram is transformed to a structural-energy diagram of the sliding friction coefficient (evolution) of the rubbing surfaces. The conclusion is made about the mutual generality of the Carter's equation and the equation of the energy balance of friction. The evolution of the structure of the rolling contact to the ideal nano-state, adequate to the state of the ideal solid, is justified. The possibility of the existence of an almost ideal thermodynamic cycle at the rolling contact and the maximum efficiency of the energy transformation is shown. In this state, we make up one mechanical (nano) quantum of the self-organized dynamic dissipative structure of the elementary friction contact with minimal contact shift (creep). This actually determines the real performance characteristics of the heavy-loaded Hertz contact of real engineering systems.

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1. INTRODUCTION

The art of the wheel and, accordingly, the rolling friction of solid elastic bodies is widely used in modern practice (gears, rolling bearings, wheel-rail system, and others [1-6]). For example, studies of rolling friction of the wheel-rail system in the railway industry are indicative [7-11]. The highly efficient operational behavior of the wheel on a heavy-loaded Hertz contact in dynamics [12-14] is associated with the special contact properties of the rolling process. Indeed,

the intense plastic deformation up to the nanostructured level of the rolling contact allows us to consider the highly activated interfacial layer as a lubricant with rheological properties (viscoelasticity) causing low friction. In general, modern studies of the rolling friction of elastic solids well reflect the fact [15] that the hysteresis losses for solid steels are extremely small (less than 0.5%), so the rolling resistance is very small ($\mu = 0,001$) and in this case lubricants play a small role in the rolling friction itself.

The purpose of the article is to reflect on the physical interrelated properties of the rolling contact: the high efficiency of the heavily loaded Hertz contact, the most developed evolution of plastic deformation, which provides the effect of quasi-elastic rolling, very low resistance to movement (friction), and the most efficient energy conversion at the contact.

In the article, the author applies his own method of friction analysis – triboergodynamics - to the process of elastic rolling of the wheel [16-20]. The structural-energy regularities of the evolution of the friction contact are considered. It is shown that the statement about the elastic rolling of the wheel, i.e., a completely reversible process, is an impossibility to distinguish the high-speed and most fully developed evolution of the plastic component of the contact rolling deformation. Elastic rolling should be considered elastic only because the evolution of the contact begins (static) and ends (dynamic) in elastic regions. Ideal evolution of tribosystem is symmetrical. Between these extreme contact states, there is an almost complete evolutionary cycle of plastic deformation, as the main mechanism of energy transformation. The process ends in an elastic region (ideal nano-structural state of contact, adequate to the state of an ideal solid) with minimal loss in the form of a single mechanical (nano) quantum (tribosubsystem). Proposed in the framework of triboergodynamics model nano-quantum damping surfaces under friction [19-21], allows to apply this model aggregate (synergistic) structural behavior of an elastic-viscous-plastic rolling contact (selforganized solid lubricant [22]) with minimal resistance (abnormally low friction) this rotation and minimal, but not zero, the loss of (a creature of “wearlessness”). These reflections allow us to consider the interpretative possibilities of analyzing the Carter’s diagram of tangential forces on the rolling contact in the context of the general friction model – stick-slip motion.

The conditional scheme (model) of the wheel contact with friction on a smooth plane in the vicinity of the material point A (Fig.1), taking into account the elastic effect of the Poisson's ratio (it is adequate to the Hans Fromm model [11] for the roll contact problem), allows us to draw conclusions about the essential properties of the rolling process as a whole.

1. The wheel-plane system under conditions of elastic equilibrium of smooth surfaces under the action of applied external forces – normal and tangential, creates an elementary equilibrium tribosystem in the area of contact of the wheel with the plane. This is a pair of elementary equilibrium roughnesses, an elastic hemisphere on the surface of the plane and an elastic hemisphere on the surface of the wheel.

2. The rolling process, which manifests itself from the very beginning of its evolution through the creation of an elementary tribosystem, must have the basic laws of its behavior that are fully adequate to the basic laws of sliding friction.

3. An ideal model of smooth Carter surfaces under elastic contact conditions when used in rolling analysis leads to the creation of a model of an elementary tribosystem for rolling realization.

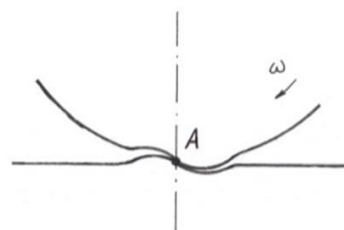


Fig. 1. Scheme of elastic equilibrium of the wheel on the plane [17].

2. BRIEF STRUCTURAL-ENERGY, BALANCE LAWS OF FRICTION

The general evolution regularities of states and properties of tribosystem in the frame of triboergodynamics are analysed. Triboergodynamics is based on our modern knowledge of friction: 1. friction is a phenomenon of resistance to relative motion between two bodies, originating at their surfaces contact area; 2. friction is the process of transformation and dissipation of energy of external movement into other kinds of energy; 3. friction is the process of elasto-plastic deformation localized in thin surface layers of rubbing materials.

Methodology of triboergodynamics [16-20] is based on the analysis method to plastic deformation of ergodynamics of deformed solids [23-27]. Ergodynamics is a synthesis to the problem of deformation most general laws of thermodynamics for non-reversible processes [23], molecular kinetics (thermo-activation

analysis) and dislocation theory [24] in their mutual, dialectical tie on the basis of a most general law of nature – the law of energy conservation at its transformations.

Within the framework of triboergodynamics [16] the model of elastic-plastic deformation of contact volumes is examined as a generalized mechanism of transformation and dissipation energy and determines essence of resistance to surfaces displacement.

Friction is a global phenomenon of transformation and dissipation of the energy of the external relative motion of rubbing bodies (for example [28,29]). It strongly obeys equation of energy balance [23,30,31] and from thermodynamic point of view [16] it is a competition of two simultaneous, interconnected and opposite tendencies of accumulating latent (potential) energy ΔU_e of various kinds of defects and damages of contact volumes structures and releasing (dissipation) energy Q due to various relaxation processes.

According to the energy balance scheme (Fig. 2) for plastic deformation and fracture [16] presented below, equations for friction work W_f , frictional force F and friction coefficient μ (without lubrication) [16] has view

$$W_f = \Delta U_e + Q = \Delta U_{e1} + \Delta U_{e2} + \Delta U_{T1} + \Delta U_{T2} + \bar{Q}_1 + \bar{Q}_2, \quad (1)$$

$$\dot{W}_f = \dot{U}_e + \dot{Q} = \dot{U}_{e1} + \dot{U}_{e2} + \dot{U}_{T1} + \dot{U}_{T2} + \dot{\bar{Q}}_1 + \dot{\bar{Q}}_2, \quad (2)$$

$$F_l = \frac{\Delta U_e}{l} + \frac{Q}{l} = \frac{\Delta U_{e1} + \Delta U_{e2}}{l} + \frac{Q_1 + Q_2}{l}, \quad (3)$$

$$F_v = \frac{\dot{U}_{e1} + \dot{U}_{e2}}{v} + \frac{\dot{Q}_1 + \dot{Q}_2}{v} = F_{\text{mechanical}} + F_{\text{molecular}}, \quad (4)$$

$$\mu_l = \frac{\Delta U_{e1} + \Delta U_{e2}}{Nl} + \frac{Q_1 + Q_2}{Nl} = \mu_{\text{adapt}} + \mu_{\text{dis}} = \mu_{\text{adapt}} + \mu_{T(\text{dis})} + \mu_{\bar{Q}(\text{dis})}, \quad (5)$$

$$\mu_v = \frac{\dot{U}_{e1} + \dot{U}_{e2}}{Nv} + \frac{\dot{Q}_1 + \dot{Q}_2}{Nv} = \mu_{\text{deformation}} + \mu_{\text{adhesion}}, \quad (6)$$

where $\Delta U_e = V_f \Delta u_e$; $Q = V_f q$; $\dot{U}_e = V_f \dot{u}_e$; $\dot{u}_e = du_e/dt$; V_f - is the deformable (friction)

volume; μ - friction coefficient; μ_{adapt} - adaptive friction coefficient; $\mu_{T(\text{dis})}$ and $\mu_{\bar{Q}(\text{dis})}$ - static and dynamical components of dissipative friction coefficient; ΔU_T - thermal component of internal energy; N - normal load; l - distance of friction. The latent energy density Δu_e is an integral parameter of tribostate and damageability (failure (Δu_e^*)).

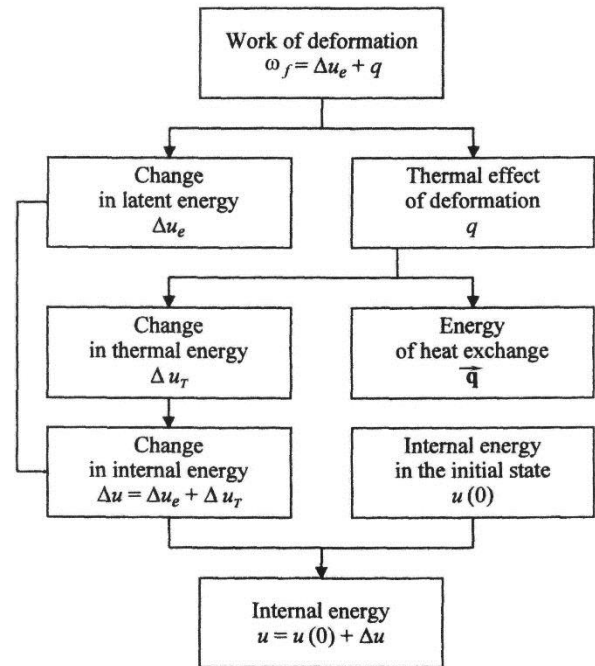


Fig. 2. Scheme of the energy balance for the plastic deformation of a solids [16].

Thus, viewed thermodynamically, the work done by friction forces W_f (the friction power \dot{W}_f), the friction force F and the friction coefficient μ may be classified conventionally into two specific components with different kinetic behavior [16,23,24]. The first component is associated with microscopic mechanisms of adaptive type and relates to the change of latent (potential) energy ($\Delta u_{e1}, \Delta u_{e2}$) of various elementary defects and damages that are generated and accumulate in the deformable volumes of materials friction pair (Fig. 3). This energy is a unique and integral characteristic of the submicro- and microstructural transformations that occur in plastically strained materials [23-27]. This energy is a measure of strain hardening and damageability of materials. The second component is associated with microscopic

mechanisms of dissipative type and relates to dynamic recovery processes in which latent energy is released and heat effect of friction (q_1, q_2) take place. This energy originates in the motion and destruction of various elementary defects of opposite signs, the egress of these defects to the surface, the healing of reversible submicroscopic discontinuities, etc. The ratios of the components Δu_{e1} and Δu_{e2} as well as q_1, q_2 of the balance vary over a wide range, depending on the physical, chemical, and structural properties of the materials that comprise the friction couple and the friction process conditions.

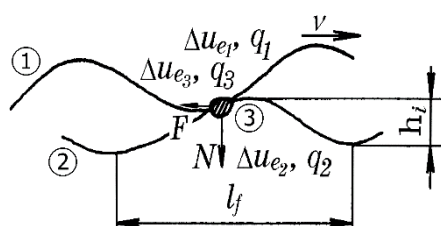


Fig. 3. Schematic view of elementary friction's contact [16,18-20]

Thus, the thermodynamic analysis of friction (plastic deformation and fracture) has led to generalized (two-term) relations (3)-(6) for the force F and coefficient of friction μ , which agrees with current concepts of the nature of friction - molecular-mechanical [32] and deformation-adhesion [15]. The author believes that it is more correct to talk about the adaptive-dissipative nature of friction (5).

Relationships (1)-(6) which generalize the mechanism of energy dissipation at friction allow to classify the tribosystem states. According to ergodynamics of deformed solids (relationships $\Delta u = \Delta u_e + \Delta u_T$ and $q = \Delta u_T + \bar{q}$) and equations (1)-(6), all exhibitions of friction and wear may be reduced conventionally at least to two basically different states: the first state defines all types of damage and wear, the second — the so-called "wearless" condition. The state of damage and wear is characterized by the components of energy balance (1)-(6), which are responsible for accumulation of internal energy in deformed volumes $\Delta u = \Delta u_{e1} + \Delta u_{e2} + \Delta u_{T1} + \Delta u_{T2}$, i.e. the process is irreversible. The "wearlessness" state is characterized by the components responsible for dynamic dissipation (reversibility) of strain energy

into elastic and structural dissipated energy of friction contact $\bar{q} = \bar{q}_1 + \bar{q}_2$.

In its turn, the first state may be classified depending on the relation between potential Δu_e and kinetic Δu_T components of internal energy. It is subdivided conventionally into mechanical damage and wear (due to so-called structure activation) and thermal damage and wear (due to thermal activation). For instance, let the thermal component of internal energy Δu_T be equal to zero ($\Delta u_T = 0$) and the internal energy variation at damage and wear be defined only by variation of the potential Δu_e ($\Delta u = \Delta u_e$) component. Then, the mechanical damage and wear with brittle fracture of surfaces take place. On the contrary, if we have $\Delta u_e = 0$ ($\Delta u = \Delta u_T$), then the thermal damage and wear with ductile fracture of surfaces take place. All the intermediate values of the components are associated with quasi-brittle or quasi-ductile fracture of solids.

In the most general case, the energy balance at dry friction (1) should be written as

$$W_f = \Delta U_{e1} + \Delta U_{e2} + \Delta U_{e3} + Q_1 + Q_2 + Q_3 \quad (7)$$

In the special case, where the friction is localized into volume of the "third body" [32] (secondary structures [33]) equation (7) develops into

$$W_f = \Delta U_{e3} + \bar{Q}_3 \quad (8)$$

According to thermodynamic theory of strength [23,25], the damageability parameter and the fracture criterion are defined in terms of the internal energy density u accumulated within the strained element of a solid body. A solid body is assumed to suffer fracture if the internal energy density has reached a critical value u^* in at least a single macrovolume that is responsible for fracture.

According to thermodynamic theory of strength [23,25] too, the true, structure process parameter should be related to the portion of the accumulated plastic deformation that is responsible for strain hardening.

With this in mind, if we neglect the heat effect Q of friction, one will infer [16] from the thermodynamic analysis of friction of equations

(1)-(6) that the Amontons (Leonardo da Vinci) friction coefficient is

$$\mu = \frac{\Delta U_e}{\mu^* N l} = \frac{F}{N}; \quad F = \frac{\Delta U_e}{l}; \quad Q \cong 0, \quad \mu^* = 1. \quad (9)$$

Consequently, the coefficient of friction has a very deep physical sense. On the one hand, it is the parameter which generally characterizes the resistance of relative displacement (movement) of surfaces, for it reflects the portion of energy, which «is done by friction away» as accumulated latent energy ΔU_e , by relation to parameter of external

forces work $\mu^* N l$ (energy of external relative movement). On the other hand, it is the generalized characteristic of damage, for it is defined of the latent energy density Δu_e as integral characteristic of the structure defectiveness measure, because this energy is the generalized parameter of damage. Here too, coefficient of friction generally reflects the structural order (disorder) of deforming contact volume, since the parameter $\Delta U_e = \Delta u_e V_f$ is defined of the energy of defects and damages of different types, that are accumulated into contact volumes V_f solids.

Therefore, coefficient of friction is a true and generalized parameter of tribosystem state. From this conclusion we can say that the analysis of the evolution of the states of a tribosystem is primarily an analysis of the latent deformation energy accumulated within the contact friction volumes.

Tribosystem evolution presented as a diagram (Fig.4), has adaptive-dissipative character (5) and reflects competitive (dialectical) nature of friction. Evolution curve has a set of principal points (1-5) of transitive states of tribosystem, which strongly obeys a balance principle of friction [16,21]. The most characteristic areas between these points reflect general properties of its non-linear dynamics.

So, in the diagram (Fig.4) it is possible to see the following conventionally designated points and stages: 0-1 – a stage of static friction and strain strength hardening; 1 – a point of ultimate strain hardening; 1-2 – a stage of pumping of excess energy; 2 – point of highly excited state - seizure and transition of external friction to internal (critical instability); 2-3 – a stage of forming dissipation structures (formation of heat

fluctuation in friction volume); 3 – a point of minimum compatibility (maximum frictionness); 1-2-3 – a stage of selforganization; 3-4' – a stage of compatibility; 4 – a point of wearlessness (anormal-low friction); 5 – a point of thermal seizure.

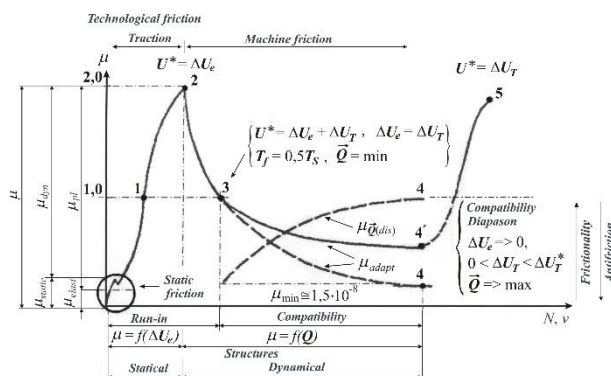


Fig. 4. Structural-energy diagram of the evolution of rubbing surfaces [16,18-22]. Marking on the axes: N, v – load and speed; $\mu_{static}, \mu_{dyn}, \mu_{elast}, \mu_{pl}$ – static, dynamic, elastic, plastic coefficients of friction; T_f, T_s – flash point in the contact volume of friction in p. 3 and the melting point.

An ideal evolution of tribosystem is symmetrical. The process starts and finishes within areas of elastic behavior. A plastic maximum (a superactivated condition) exists between them as a condition of selforganisation and adaptation.

In the most general case evolution (adaptation) regularities of tribosystems may be presented as a 2-stage (Fig.4). At the first stage (0-2) of adaptation the evolution of friction contact rushes to form

some critical volume of friction V_f^* (point 2). It is elementary tribosystem that is the elementary and self-sufficient energy transformer. The first stage – latent energy density growth Δu_e to a limited magnitude Δu_e^* within critical friction volume V_f^* .

This friction volume V_f^* is constant at the second stage of evolution, but here it is evolutionary developed owing to structural transformation; by this one may realize wide spectrum of compatibility friction structures (Fig. 4).

The second stage (2-4) – structural transformation of critical friction volume (elementary tribosystem) V_f^* into adaptive V_{adapt} and dissipative V_{dis} volumes. The limit (point 4) of this stage is

characterized by a full transformation of adaptive critical volume V_{adapt}^* into V_{dis}^* dissipative.

The volumes mentioned above characterize different regularities of transforming energy of outer mechanical movement at friction. Adaptive volume V_{adapt} is connected with non-reversible absorption of deformation energy. It is in this volume where latent deformation energy Δu_e accumulates and where the centres of destruction initially emerge (birth). Dissipative volume V_{dis} is capable of reversible transformation (dissipate) of outer movement energy. It doesn't accumulate latent deformation energy owing to reversible elastic-viscous-plastic deformation.

Suggested theoretical and calculation assessments [16, 18-22] showed that dissipative friction volume performs reversible elastic energy transformation of outer mechanical movement with density \bar{q}^* equal to critical density of latent energy u_e^* .

Culmination of tribosystem evolution is its final and limited condition of point 4 – a state of anomalously low friction and wearlessness (maximum efficient).

Calculation show [16] that at an ideal tribosystem evolution an adaptive (Amontons) friction coefficient μ_{adapt} in a point 2 of a diagram falls abruptly down, reaching in a point 4 the value of elastic friction coefficient μ_{elast} . For point 4 of compatibility area 3-4 an equation of energy balance (5) showed to be put in the following way:

$$\begin{aligned} \mu_{adapt} &= \mu^* - \mu_{dis} = \\ &= 1 - \mu_{dis} = \mu_{plast} = 0 = \mu_{elast}; \quad \mu^* = 1,0. \end{aligned} \quad (10)$$

Thus, point 4 (Fig. 4) stands for an ideal evolution of contact friction volume a condition of ideal elastic-viscous-plastic deformation. Equation (10) shows as a matter of fact exactly it, i.e. Amontons friction coefficient μ_{adapt} being in its essence plastic friction coefficient μ_{plast} has a minimum value equal to zero. It follows then, that plastic friction became elastic with friction coefficient μ_{elast} . It means that plastic deformation of contact volume friction is implemented with the maximum dynamic dissipation ($\bar{Q} = \max$) of accumulated

latent energy. That is why the value of accumulated energy in point 4 is equal to zero ($\Delta U_e = 0$). This fact proves an ideal condition at most full evolution of contact volume. From the physics point of view this state may be explained by the full dissipation of accumulated energy ΔU_e^* in point 2 and by newly emerged structures of point 4 in the form of elastic energy of interaction between them (dynamic dissipation energy \bar{Q}^*). Here $\mu_{dis} = 1,0$. The structural elements themselves are defectlessness - $\mu_{adapt} = 0$, and friction is elastic - $\mu = \mu_{elast}$.

It has been demonstrated [16] that value of minimum adaptive friction volume V_{adapt}^{\min} corresponding to the zero meaning of plastic friction component μ_{adapt} is not equal to zero, but is equal to some minimum structural element of deformed solid body - the mechanical (nano) quantum.

This mechanical quantum, as a unique elementary nanostructure, is the result of the most ideal evolution of the elementary tribosystem-the friction contact (see below).

3. ROLLING MODEL BASED ON THE ANALYSIS OF THE CARTER DIAGRAM OF TANGENTIAL FORCES ON THE CONTACT

To move the center of the wheel and start the rolling process, a certain limit equilibrium, elastic state is necessary. The elastic state of the wheel contact creates an instability that precedes the beginning of the rolling process – turning around the instantaneous center of rotation A and moving the wheel's center of gravity from the point of initial equilibrium to a new position. The reason for rolling is the fact that plastic deformation occurs on the contact.

The view of the Carter's diagram [7] of changes in tangential forces at the rolling contact resembles the curve of stretching test metals to the state of destruction (loss of stability). In the Carter's diagram [7], taking into account the stretching test diagram of materials, the following stages can be distinguished (Fig. 5): 1 – area of elastic deformation (or elastic displacement on the contact); 2 – area of plastic

deformation by the mechanism of conservative dislocation sliding (strain hardening); 3 - respectively, the site of the neck formation; this is the localization of the process (concentrated deformation) and the growth of stresses in a certain hydrostatic region of deformation (scheme of all-round irregular compression); point A is the point of the critical state preceding failure or loss of stability in this local critical volume of deformation.

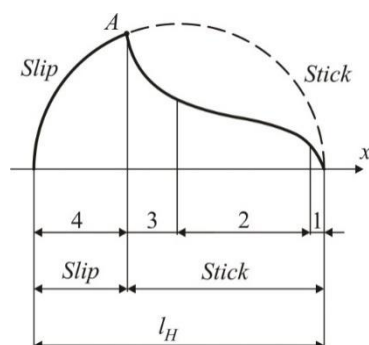


Fig. 5. To interpretation the Carter diagram of tangential rolling contact forces.

In general, Carter calls this entire area the "stick" area. This area can also be designated as the area of joint elastic-plastic deformation of the contact volumes of the wheel and the plane (rail). 4 - this is the Carter area of slip. According to the stretching diagram of metallic materials, this area can correspond to the behavior of metals in the area of neck formation - failure (stress relaxation) - either by a brittle sudden fracture mechanism and the separation of the sample into parts, or by a viscous fracture mechanism.

It is noteworthy that in the Carter tangential force diagram, the drop in tangential stresses occurs up to zero - to the initial elastic area. This indicates the possible state of the deformable contact volume in the second rolling stage - here there is a drop in the sliding friction resistance to the maximum level - to the state of abnormally low, almost zero friction. The fact that the stresses here do not drop instantly indicates that there is no brittle separation of the contact area, and there is a loss of stability in the form of relaxation of internal stresses of pre-hardening and relaxation (restructuring) of the structure of the contact volume. It can be assumed that in this region, the stresses fall due to the

conversion of the energy accumulated at the first stick stage into heat energy and further dissipative structure formation. Here, the stress drops down to zero, i.e., the absolutely elastic state - the ideal dissipative viscoplastic behavior of the contact volume with minimal friction (viscosity). This state corresponds to a point 4 in accordance with the structural-energy diagram of sliding friction proposed in the framework of triboergodynamics [16-22] (Fig. 4).

4. THERMODYNAMIC INTERPRETATION OF THE CARTER DIAGRAM OF TANGENTIAL FORCES ON THE ROLLING CONTACT

At the wheel contact (Fig.6), the friction force (coefficient of friction) obeys the equation of the energy balance of friction [17]. The equations of the energy balance of friction taking into account the regularities of accumulation have the form:

$$\tau = \tau_{static} + \tau_{dyn} = \tau_{elast} + \frac{\Delta U'_e}{A_H l_H} + \frac{\Delta U''_e}{A_H l_H} + \frac{\Delta U_T}{A_H l_H} + \frac{\bar{Q}}{A_H l_H}, \quad (11)$$

$$\begin{aligned} \mu &= \mu_{static} + \mu_{dyn} = \\ &= \mu_{elast} + \frac{\Delta U'_e}{p_H A_H l_H} + \frac{\Delta U''_e}{p_H A_H l_H} + \\ &\quad + \frac{\Delta U_T}{p_H A_H l_H} + \frac{\bar{Q}}{p_H A_H l_H} = \\ &= \mu_{elast} + \frac{\Delta U'_e}{N_H l_H} + \frac{\Delta U''_e}{N_H l_H} + \frac{\Delta U_T}{N_H l_H} + \frac{\bar{Q}}{N_H l_H}, \quad (12) \end{aligned}$$

where $p_H \cdot A_H \cdot l_H$ - respectively, the specific normal load, area and length of the Hertz rolling contact; $\Delta U'_e$ and $\Delta U''_e$ - components of changes in latent (potential) energy that differ in the patterns of their accumulation; the first is related to dislocation-type mechanisms (strain hardening), and the second - with mechanisms of the vacancy type (super-activated state); ΔU_T - change in the thermal (kinetic) component of internal energy; \bar{Q} - power of dynamic energy dissipation.

In Fig. 6, points 1,2,3,4 correspond to similar points in the diagram in Fig. 4.

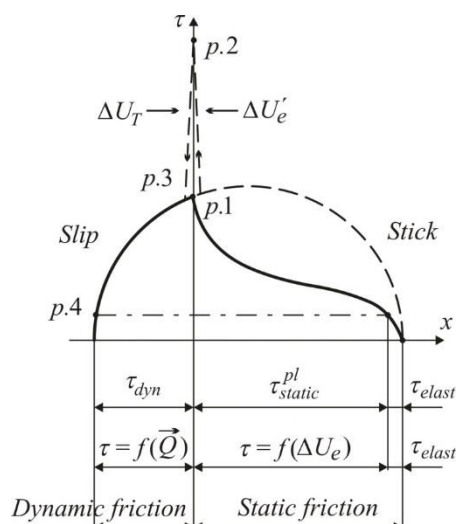


Fig. 6. Energy interpretation of the Carter diagram of tangential forces on the rolling contact.

5. CONVERT THE DIAGRAM OF THE CARTER TANGENTIAL FORCES AT THE CONTACT ROLLING TO THE DIAGRAM OF THE COEFFICIENT OF FRICTION AT THE ROLLING CONTACT

We turn the Carter diagram to 180° , which does not change its fundamental essence. Divide the values of the tangent stresses τ of the diagram line by the corresponding normal stresses σ described by the radius of this diagram.

The appearance of this transformed diagram (Fig. 7) characterizes the regularities of the evolution of elastic-plastic deformation of the contact volume, which provides in general equilibrium elastic state of the rolling contact.

In the diagram (Fig. 7), the friction coefficient curve has two most characteristic stages. The first one shows the increase of the coefficient of friction μ_{adapt} to a critical state at the point C, and the second one shows the constancy of the coefficient of friction μ_{diss} with a characteristic balance value equal to one. Point C, respectively, divides the two stages of the evolution of the elementary rolling contact (tribosystem) into the static rolling contact and the dynamics – the actual rolling process – the rotation of the wheel around the instantaneous center of rotation.

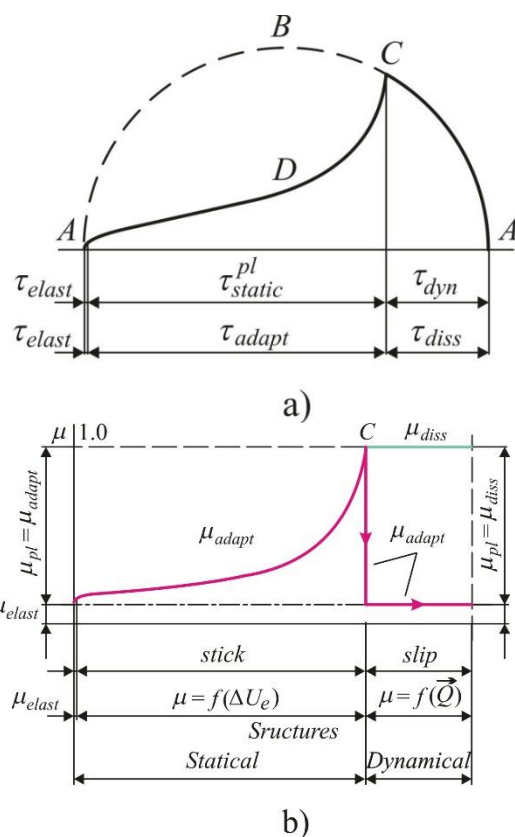


Fig. 7. Transformation of the Carter diagram of tangential forces on the rolling contact (a) to the diagram of the coefficient of friction on the rolling contact (b).

Accordingly, the appearance of this transformed diagram fully corresponds to the structural-energy diagram (Fig. 4) of the evolution of rubbing surfaces under sliding friction.

In accordance with the energy balance equation for the coefficient of friction on the rolling contact, we have the most complete transformation-energy cycle of the evolution of this contact. At the first stage, the contact accumulates energy ΔU_e to a critical level ΔU_e^* at the point C, and then, at the second stage, this energy is released into the energy $Q = \bar{Q} = \Delta U_e^*$ of dynamic dissipation.

In this case, the level of accumulated energy falls to a minimum level equal to the level of the initial elasticity, i.e., the plastic dissipative component of friction at the second stage characterizes the quasi-ideal viscoplastic state of contact with a minimum coefficient of viscosity (friction) – rotational modes of developed plastic deformation-abnormally low friction.

Therefore, the coefficient of friction equal to one in the second section of the diagram is the essence of the dissipative coefficient of friction. The adaptive (Amonton) coefficient of friction μ_{adapt} at a point C drops sharply, reaching the value of the elastic coefficient of friction μ_{elast} , i.e. (see (10)).

Consequently, the equilibrium rolling process begins in the elastic region and ends in the elastic region, realizing the full cycle of evolutionary adaptation of the contact structure. Therefore, in general, we have an equilibrium process of rolling the wheel at Hertz contact, in which between two elastic regions there is an intermediate highly activated equilibrium state (equilibrium away from the equilibrium state) with the maximum possible degree of reversibility.

As a result of this evolution of the elementary rolling contact, there is a minimal loss - wear in the form of a single mechanical quantum. As shown in the framework of the model of dynamic dissipation under friction [16], for any elastic critical interaction, including during rupture (destruction), when the formation of surfaces takes place, it is necessary to reversibly expend or reversibly return to the environment an amount of energy equal in value to the energy of one mechanical quantum.

At the point C, equating the expressions for the coefficients of friction of the two stages - $\mu_1 = \mu_2$, we obtain the condition of the contact state for elastic rolling in the presence of a plastic component, namely - $\mu_1 = \mu_2 = \mu_{elast}$, since

$$\Delta U_e = \Delta U_e^* = -Q = -\bar{Q}.$$

Thus, despite the presence of a plastic contact state during rolling, in general, the rolling contact state can be elastic. This is due to the possibility of implementing a full evolutionary cycle of motion energy transformation on the rolling contact, which actually characterizes the state of the most fully developed contact deformation as the main mechanism of motion energy transformation.

6. MECHANICAL (NANO) QUANTUM AND AN IDEAL DYNAMIC DISSIPATIVE FRICTION STRUCTURE

As a result of the most complete evolution of the elementary tribosystem (Fig. 4, point 4), a unique contact nanostructure is formed, which is based on a mechanical (nano) quantum (Fig. 8). This elementary nanostructure is most interesting from the point of view of analyzing the state and properties of dissipative dynamic friction structures.

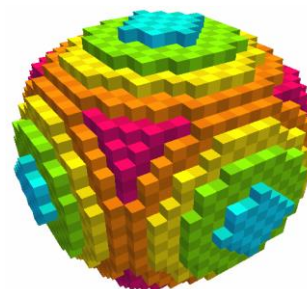


Fig. 8. Model of an ideal crystal of the elementary nanostructure of the friction contact [16,22,34].

A mechanical quantum [16,22,34] is the minimum number of atoms capable of providing such a configuration distribution (structure) that has the property of reversibly perceiving and dissipating (returning) the energy of external mechanical motion (impact). It also represents the smallest structural formation under conditions of plastic deformation and is formed when the tribosystem (deformable volume) passes through an extremely activated (critical) state (area 1-2-3 on Fig.4) due to the development of self-organizing processes of tribosystem adaptation. This is a structural attractor of the limit cycle for plastic deformation.

The mutual rotational-vibrational motion of these mechanical quanta relative to each other within an elementary tribosystem determines the state of the most perfect dynamic dissipative friction structure. In fact, such a state is described by the equation of state quasi-perfect solid [16,22,35], the interaction between the elements of the structure (mechanical quanta) is minimized - a state of perfect elasticity quasi-viscous flow. The calculated coefficient of friction between mechanical quanta is approximately - 10^{-8} .

The contact volume of friction (an elementary tribosystem) with an equilibrium size of about 2.85 microns contains 65659969 mechanical

(nano) quanta [16,22,34]. Actually, a thin self-organized layer of dynamic dissipative friction structures is formed. This is the effect of perfect structural scattering of stored energy. Inside this layer (the third body - solid lubricant), the friction process is implemented with a minimum resistance of the order of $\mu_{adapt} = 0,0022$ [19,22] and the least wear – one mechanical (nano) quantum of losses per contact load act.

7. EQUATION OF THE ENERGY BALANCE OF THE ROLLING PROCESS

It can be shown that the two-stage rolling process (stick and slip), from the point of view of the general analysis of friction and from the point of view of triboergodynamics [16,17], corresponds to the general dual laws of external friction and reflects the adaptive component of the friction process at the first stage, and the dissipative one at the second stage.

Therefore, for the first stage of the rolling process, the stage when the wheel surface comes into contact (stick), you can write

$$T \cdot v_R = \mu \cdot N \cdot v_R = \dot{U}_e \quad (13)$$

Here T is the pulling force on the wheel friction contact; $v_R = \omega \cdot R$ - the speed at which the wheel makes contact with the rail; ω, R - angular velocity of the wheel and its radius; \dot{U}_e - the rate of accumulation of latent (potential) energy by the deformable contact.

Accordingly, for the right side of the Carter diagram (Fig. 5,6), we have one coefficient of friction

$$\mu = \mu_{adapt} = \frac{\dot{U}_e}{N \cdot v_R} = 1 \quad (14)$$

For the second stage of the rolling process, the slip stage, we write

$$T_T \cdot v_0 = \mu'_R \cdot N \cdot v_0 = \dot{Q} \quad (15)$$

Here T_T is the tractive force in the center of the wheel; v_0 - speed of the carriage (wheel center);

\dot{Q} - power of dynamic energy dissipation by contact.

Actually, the second stage of the rolling process is a dynamic dissipative state of contact, the movement of the center of mass of the wheel with sliding (creep).

Accordingly, the second stage of friction is characterized by its own coefficient of friction, which, in fact, is the coefficient of sliding friction, but the dissipative coefficient of friction

$$\mu' = \frac{\dot{Q}}{N \cdot v_0} = \mu_{diss} = 1 \quad (16)$$

In an ideal rolling process, the amount of energy ΔU_e accumulated by the volume of friction at the first stage of rolling is equal to the amount of energy \bar{Q} performing rolling work at the second stage ($\Delta U_e = -\bar{Q}$), or the rate of energy \dot{U}_e accumulation is equal to the rate of its \dot{Q} release

$$\dot{U}_e = -\dot{Q} \quad (17)$$

Thus, we can have:

$$T \cdot v_R = T_T \cdot v_0; \quad (18)$$

$$\mu \cdot N \cdot v_R = \mu' \cdot N \cdot v_0 \quad (19)$$

and

$$\frac{\mu}{\mu'} = \frac{v_0}{v_R} \quad (20)$$

In the real rolling process, taking into account that the speed difference $v_R - v_0$ is not equal to zero and this is the amount v_{SL}^* of slippage (creep) of surfaces, we can write the following chain of transformations of the ratio (20):

$$\begin{aligned} \frac{\mu}{\mu'} &= \frac{\mu}{\mu_{diss}} = \frac{v_0}{v_0 + v_{SL}^*}; \\ \mu \cdot (v_0 + v_{SL}^*) &= \mu' \cdot v_0; \\ \mu' &= \mu \cdot \left(1 + \frac{v_{SL}^*}{v_0} \right) = \mu_{diss} \end{aligned} \quad (21)$$

In equation (21), we perform the following mathematical procedure in parentheses:

$$\mu' = \mu \cdot \left(1 - \left(-\frac{v_{SL}^*}{v_0} \right) \right) = \mu_{diss}$$

Multiply both sides of the last ratio by the normal load N , and denote the negative speed ratio in parentheses by the parameter ξ

$$\mu' \cdot N = \mu \cdot N \cdot \left(1 - \left(-\frac{v_{SL}^*}{v_0} \right) \right) = (1 - \xi) = T_T \quad (22)$$

And as a result, we get the following equations for the rolling process of the wheel:

$$T_T = T \cdot (1 - \xi), \quad (23)$$

$$T_T = T - T \cdot \xi, \quad (24)$$

which, in fact, are the energy balance equations of the rolling process.

The resulting equations are similar to the Carter's equation [7]

$$T_T = \mu \cdot N \cdot \left(1 - \left(1 - \frac{\xi_X \cdot E^*}{2\mu \cdot p_H} \right)^2 \right), \quad (25)$$

If the value $\left(1 - \frac{\xi_X \cdot E^*}{2\mu \cdot p_H} \right)^2$ in this equation is denoted by ξ a letter, we get the equation of the energy balance of the rolling process (23):

$$T_T = \mu \cdot N \cdot (1 - \xi). \quad (26)$$

Thus, there is a complete correspondence between the obtained equation of the energy balance of the rolling process (23) and the Carter's equation [7]. Moreover, it should be noted that the thermodynamic analysis of the sliding friction process leads to the Carter equation.

Here it is necessary to note one fundamental circumstance of an essential property. If we use equations (21) and (22) to analyze the Carter curve, we can get a somewhat contradictory result. The fact is that these equations are obtained from the analysis taking into account the dissipative coefficient of friction - $\mu'_R = \mu_{diss}$. To better match the Carter curve, equations (21) and (22) should be presented taking into account the adaptive coefficient of friction -

$\mu'_R = \mu_{dis} + \mu_{adapt}$ at the second stage of rolling, converting equality (20). As a result, we get equations similar to equations (21) - (24), but with an essential difference. Namely:

$$\mu'_R = \mu \cdot (1 - \xi) = \mu_{dis} + \mu_{adapt} \quad (27)$$

After making the transformations [4], we get the following record forms:

$$\mu \cdot (1 - \xi) = \mu_{diss} + \mu_{adapt} = 1; \quad (28)$$

$$\mu \cdot (1 - \xi) = \mu_{diss} + \mu_{elast} = 1; \quad (29)$$

$$\mu N \cdot (1 - \xi) = T_{diss} + T_{adapt} = T_T = N; \quad (30)$$

$$\mu N \cdot (1 - \xi) = T_{diss} + T_{elast} = T_T = N. \quad (31)$$

In the left part of these equations, there is a coefficient of friction $\mu = \mu_{adapt} \rightarrow 1,0$ that characterizes the first stage of the rolling process - creating prerequisites for shifting the center of mass of the wheel, and in the right part there are coefficients of friction $\mu_{adapt} = \mu_{elast}$ (see (10)) and $\mu_{diss} \cong 1,0$, which characterize the second stage of the rolling process, the actual rolling process itself (moving the center of mass of the wheel).

From the relations for traction (30), (31), the following conclusions follow:

1. The maximum traction force T_T is equal to the normal load - N ;

2. Traction consists of two components: T_{adapt} - coupling, which determines the resistance to movement and pulling T_{diss} - inertia component of traction.

These equations should be used to analyze the Carter curve and consider the patterns and causes of changes in the traction force when the speed of movement of the crew (carriage) changes. Equations (21) - (24) in this analysis will act as auxiliary, allowing you to see the second side of the dual phenomenon of the friction process and the rolling process. Together ((21) and (27)) these two ways of writing the energy balance equations of friction (rolling) allow us to understand the essence of sliding friction (creep) on the rolling contact.

8. ON THE THERMODYNAMIC CYCLE OF THE ROLLING PROCESS UNDER ELASTIC CONTACT

The phenomenology of the Carter equation for wheel rolling under conditions of elastic reversible contact and the Carter's diagram [7] of tangential stresses on the Hertz rolling contact should be analyzed using thermodynamic concepts of the process as a thermodynamic cycle.

The analysis of the Carter's diagram and the analysis of the Carter's equation for the elastic behavior of the contact, performed above, allow us to present the Carter's diagram as a diagram of the thermodynamic cycle of energy transformations on the contact and give it an essential thermodynamic assessment.

The thermodynamic cycle of the rolling process, taking into account the Carter diagram, can be represented in Fig. 9. This figure shows that the Carter process of elastic rolling of the wheel is completely reversible and in this case we have no contact losses. There is an absolutely perfect and reversible thermodynamic cycle. The process starts in the elastic region (point 0) and ends in the same elastic region (point 4). Point 4 lies at the level of point 0 and, in fact, we return again to the original point 0.

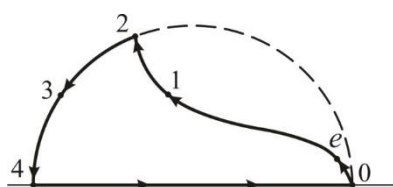


Fig. 9. Scheme of imaginary, ideal and reversible thermodynamic cycle in the crankcase diagram of elastic rolling contact.

All the energy accumulated at the first stage (path 0-e-1-2) by the rolling contact system is completely converted into the released energy of the system (path 2-3-4). As a result, there is no energy loss, and the system works perfectly - Perpetuum mobile. Here it is quite possible to apply the molecular model of friction of G.A. Tomlinson [36,37], who believed that the forces of attraction between surfaces are negligible and the load is completely balanced by the repulsive forces of oscillating and energy-dissipating molecules.

On the other hand, the model of a perfectly flat surface of the Carter, but at the same time operating under elastic contact conditions, is a surface model that we can imitate from the point of view of nano-quantum representations of dynamic, structural energy scattering [16,19-22], as the ground state of the system in which all mechanical quanta are directed against the field. In such a state, the system cannot give energy to any other system simply because it (the system) does not accumulate energy in this state [16]. Almost a complete analogy with the Tomlinson model, except, of course, that in the nano-quantum model of surface damping, one quantum is still able to behave differently from the others – it is lost (wear standard [35]). And this is the whole point of the difference between the real and the ideal.

Thus, the thermodynamic cycle of Fig. 9 is the ideal representation of Carter, taking into account his formula and his diagram, which, for example, can be interpreted by the Tomlinson friction model.

What do we actually have in real practice? Real practice, together with the classical thermodynamics of irreversible processes, gives a more specific answer regarding the degree of reversibility of thermodynamic cycles. In real practice, we are dealing with a certain degree of irreversibility or pair-related reversibility with it.

The scheme of the real minimally irreversible rolling process of the wheel under the conditions of elastic contact deformation can be considered in Fig. 10. In accordance with the structural-energy diagram of rubbing surfaces [16] and the analysis of the minimum state of friction under the conditions of plastic deformation of the tribocontact [19,22,34], the process of energy transformation on the contact does not have complete reversibility. The process ends at a certain point 4, which characterizes the minimum (zero) value of the adaptive component of friction, but which is equal to the value of the initial elasticity-the level e-4.

The nano-quantum model of dissipative damping of surfaces [16,21] allowed us to show that this minimum value is one mechanical (nano) quantum. In any real thermodynamic process, at its very beginning, we must spend (lose) some minimum amount of prerogative (energy), which is equal to the amount of energy returned from the operating system to the external environment, i.e., to the activating factor.

As a result, this is the principle of the impossibility of creating an absolutely perfect process.

The actual scheme of the real minimally irreversible (maximally reversible) process of such energy transformation is shown in Fig. 10. The process ends at point 4, which does not lie at the initial zero level of point 0. The process ends with a minimum loss equal to the value of the initial elastic energy that was spent on activating the transformation process itself. The value of this lost energy in one elementary elastic (quasi-ideal) rolling act is equal to the energy of one mechanical quantum – the energy potential of one J. Gibbs cell [38].

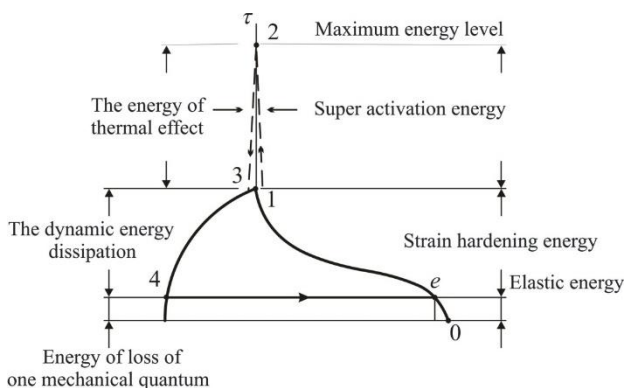


Fig. 10. Diagram of a real irreversible thermodynamic cycle in the Carter diagram of an elastic rolling contact based on the energy model of friction.

Consequently, the minimal, elastic behavior of surfaces under rolling friction is characterized only by the minimum degree of irreversibility or actually the maximum degree of reversibility in the framework of classical concepts of practice and thermodynamics about the impossibility of creating a Perpetuum mobile.

9. THE PRINCIPLE OF CRITICAL ROLLING SPEED OF THE WHEEL

The limit of this speed is determined by the principle of filling the entire elementary nominal friction area of the sliding system with elementary tribosystems (the equilibrium roughness) that dampen the process [17]. Above this speed of movement of the crew (wagon), the tribosystem will be completely unloaded, the wheel will detach from the rail surface, since the principle of minimum resistance to movement (the principle of one elementary tribosystem or the principle of

irreversibility) will be violated. In this case, all the mechanical quanta of the elementary tribosystem will repel the wheel, but there will not be a single quantum that activates the process of maintaining the system in an excited state. After the wheel is detached from the rail, the friction system ceases to activate (exist) and, of course, work.

The wheel rotates freely (cleanly) and in the next moment falls on the rail with a strong seizure, scoring and complete melting of the contact zone. This is the ultimate-emergency situation.

The calculation will be made in the following order for the wheel. The elementary nominal size of the dry friction tribosystem is known. It is equal, in accordance with [16], approximately, to the area with the size of the base length with roughness of the equilibrium level (about 1 centimeter). An elementary nominal area can simultaneously accommodate and operate $n_{TS}^* = 0,65659969 \cdot 10^8$ elementary tribosystems. Each elementary tribosystem has a size (for the spherical equilibrium roughness model) of $D_{ITS} = 2,85 \cdot 10^{-6} m$ [39] and is capable of providing the rolling path of the wheel in an elementary rolling act along the length of this tribosystem. Thus, if all the elementary tribosystems work in a unit of time over the entire elementary nominal area of dry friction, then the path traversed by the wheel in a unit of time is equal to:

$$L_{\sum TS}^{\max} = D_{ITS} \cdot n_{TS}^* = 2,85 \cdot 10^{-6} \cdot 0,65659969 \cdot 10^8 = 187,13 m/s = 673,67 km/h.$$

This result is close to the current speed 574,8 km/h achieved by TGV (France).

10. CONCLUSION

As shown by the above reflections on the nature of rolling, the rolling contact itself has fundamental evolutionary laws. The most developed rolling process is almost elastic, due to the most fully developed evolution of plastic contact deformation. An almost ideal thermodynamic cycle of energy transformation is realized at the rolling contact from the point of view of the structural-energy interpretation of the elastic-plastic friction deformation process. The friction loss is very small - one mechanical

(nano) quantum of the self-organized dynamic dissipative structure of an elementary friction contact consisting of about $0,65659969 \cdot 10^8$ mechanical quanta. At the first stage of the rolling process, a pulling force (coupling) and a friction force on the contact are created, as a result of the accumulated potential energy of defects in the contact structure. At the second stage, the accumulated energy is released and dissipated as elastic surface energy to the newly formed contact structural elements (spherical theoretical crystals – mechanical quanta). Then these nano-quanta work together, perform mutual rotations - the effect of the Rayleigh-Benard cell type. In essence mechanical quanta are molecules of a solid body. The result of rotations is an elastic shift inside the contact volume (support for external relative motion at the internal molecular level) and the wheel's center of mass is moved (an elementary rolling act) with minimal loss in the form of a single mechanical quantum and minimal contact shift (creep).

High self-organized properties of the dissipative structure of the friction contact determine the actual performance characteristics of the Hertz contact of real engineering systems.

Such a maximum nano-structuring and quasi-elastic state of the rolling contact with a minimum coefficient of friction naturally determines no need for lubrication for the rolling process itself. Indeed, lubrication is required [15] only to reduce friction between, for example, rolling elements in bearings and separators, as well as to reduce surface corrosion and thermal cycling.

REFERENCES

- [1] I.V. Kragelsky, M.N. Dobychin, V.S. Kombatov, *Friction and Wear: Calculation Methods*, Pergamon, 1982, doi: [10.1016/C2013-0-03333-6](https://doi.org/10.1016/C2013-0-03333-6)
- [2] K.L. Johnson, *Contact Mechanics*, Cambridge University Press, 1985, doi: [10.1017/CBO9781139171731](https://doi.org/10.1017/CBO9781139171731)
- [3] A.V. Chichinadze, E.D. Braun, N.A. Bouschet, *Fundamentals of Tribology (Friction, Wear, Lubrication)*, Moscow: Mashinostroenie, 2001. (in Russian)
- [4] R. Bassani, *Tribology*, Pisa: University Press, 2013.
- [5] V.L. Popov, *Mechanics of Contact Interaction and Physics of Friction. From Nanotribology to Earthquake Dynamics*, Moscow: Fizmatlit, 2013. (in Russian)
- [6] A.P. Ivanov, *Rolling Friction*, Doklady Physics, vol. 64, pp. 129-133, 2019, doi: [10.31857/S0869-56524853295-299](https://doi.org/10.31857/S0869-56524853295-299)
- [7] F.M. Carter, *On the action of a locomotive driving wheel*, Proceedings of the Royal Society of London, Series A, vol. 112, pp. 151-157, 1926, doi: [10.1098/rspa.1926.0100](https://doi.org/10.1098/rspa.1926.0100)
- [8] W.J. Harris, S.M. Zacharov, J. Lungren, H.Tournay, W.Ebersöhn, *Guidelines To Best Practices For Heavy Haul Railway Operations: Wheel And Rail Interface Issues*, International Heavy Haul Association, 2001.
- [9] F. Bucher, K. Knothe, A. Theiler, *Normal and tangential contact problem of surfaces with measured roughness*, Wear, vol. 253, iss. 1-2, pp. 204-218, 2002, doi: [10.1016/S0043-1648\(02\)00102-3](https://doi.org/10.1016/S0043-1648(02)00102-3)
- [10] F. Bucher, T. Klimpel, K.L. Johnson, *Two-dimensional normal and tangential rail/wheel contact with rough surfaces: Comparison between a Greenwood/Tripp-like stochastic analysis and results for a deterministic steady-state model*, Tribology Series, vol. 39, pp. 551-562, 2000, doi: [10.1016/S0167-8922\(01\)80138-3](https://doi.org/10.1016/S0167-8922(01)80138-3)
- [11] K. Knothe, *150 Jahre Bahntechnische Forschung In Deutschland*, ZEVrail: Zeitschrift für das gesamte System Bahn, vol. 127, iss. 4, pp. 3-4, 2003. (in German)
- [12] V.L. Popov, S.G. Psakhie, E.V. Shilko, A.I. Dmitriev, K. Knothe, F. Bucher, M. Ertz, *Friction coefficient in "rail – wheel" - contacts as a function of material and loading parameters*, Physical Mesomechanics, Vol. 5, No.3, pp. 17-24, 2002.
- [13] R.I. Popovici, *Friction in Wheel - Rail Contacts*, PhD thesis, University of Twente, Enschede Netherlands, 2010.
- [14] J.O. Yugat, J.M. Miralles, M. De los Santos, *Analytical model of wheel-rail contact force due to the passage of a railway vehicle on a curved track*, Revista Facultad de Ingenieria Universidad de Antioquia, no. 50, 2009.
- [15] F.P. Bowden, D. Tabor, *Friction: An Introduction To Tribology*, Anchor Press, 1973.
- [16] S.V. Fedorov, *The Foundations of Triboergodynamics and Physico-Chemical Prerequisites of Compatibility Theory*, Kaliningrad State Technical University Press, 2003. (in Russian)
- [17] S.V. Fedorov, *Energy Nature Of The Wheel Elastic Rolling*, Kaliningrad State Technical University Press, 2004. (in Russian)

- [18] S.V. Fedorov, *Energy Balance Of Friction And Friction Coefficient In Energetical Interpretation*, *Tribology in Industry*, vol. 37, no. 3, pp. 380-389, 2015.
- [19] S.V. Fedorov, *Structural-Energy Interpretation of the Friction*, in M.A. Chowdhury (Ed.): *Friction, Lubrication and Wear*, London, United Kingdom, IntechOpen Ltd., pp. 35-52, 2019, doi: [10.5772/intechopen.86123](https://doi.org/10.5772/intechopen.86123)
- [20] S.V. Fedorov, *Friction Energy Balance Regularities And Tribology's Nano-Structural Standard*, *COMADEM, Condition Monitoring and Diagnostic Engineering Management*, vol. 23, no. 1, pp. 13-30, 2020.
- [21] S.V. Fedorov, E. Assenova, *Synergy And Self-Organization In Tribosystem's Evolution. Energy Model of Friction*, in 9th International Conference on Tribology, Balkantrib'17, 13-15 September, 2017, Nevsehir, Turkey, IOP Conference Series: Materials Science and Engineering, vol. 295, 2018, doi: [10.1088/1757-899X/295/1/012028](https://doi.org/10.1088/1757-899X/295/1/012028)
- [22] S. Fedorov, *Selforganized nano-quantum solid lubricant*, *Tribologie und Schmierungstechnik*, vol. 63, iss. 3, pp. 5-13, 2016.
- [23] V.V. Fedorov, *Thermodynamic Aspects of Strength and Fracture of Solids*, Tashkent: Science, 1979. (in Russian)
- [24] V.V. Fedorov, *Kinetics of Damage and Fracture of Solids*, Tashkent: Science, 1985. (in Russian)
- [25] V.V. Fedorov, *Ergodynamic Concept of Failure. 1. Basic Statements of the Ergodynamics of Deformed Bodies, Criteria of Failure Ductility*, *Strength of Materials*, vol. 23, no. 8, pp. 883-889, 1991.
- [26] V.V. Fedorov, *The Ergodynamic Concept of Failure. 2. Kinetics of Crack Propagation*, *Strength of Materials*, vol. 23, no. 8, pp. 890-896, 1991.
- [27] V.V. Fedorov, *The Ergodynamic Concept of Fracture. Communication 3. Structure and Criteria of Fracture Toughness*, *Strength of Materials*, vol. 23, no. 10, pp. 1081-1085, 1991.
- [28] G. Fleischer, *40 years of evaluation of friction and wear with the help of energy density*, *Tribologie und Schmierungstechnik*, vol. 51, iss. 3, pp. 5-11, 2004. (in German)
- [29] G. Fleischer, *Tross's findings from today's perspective*, in 1st Arnold Tross Kolloquium, 10 Juni, 2005, Hamburg, Germany, pp. 215-242, 2005. (in German)
- [30] B.I. Kostetsky, Yu. I. Linnik, *Investigation of the Energy Balance Under External Metal Friction*, *Mashinovedenie*, no. 5, pp. 82-94, 1968. (in Russian)
- [31] J. Sadowski, L. Sarnowicz, *Contribution to calorimetric frictional heat measurement*, *Tribologie und Schmierungstechnik*, vol. 66, iss. 1, pp. 34-41, 2019. (in German)
- [32] I.V. Kragelskii, M.N. Dobyclun, V.S. Kombalov, *Friction and Wear Calculation Methods*, Moscow: Mechanical Engineering, 1977. (in Russian)
- [33] B.I. Kostetskii, *Friction, Lubrication and Wear in Machines*, Kiev: Technics, 1970. (in Russian)
- [34] S.V. Fedorov, *The Mechanical Quantum of Dissipative Friction Structures is the Elementary Tribonanostructure*, in 4th World Tribology Congress, 6-11 September, 2009, Kyoto, Japan, Japanese Society of Tribologists, p. 926.
- [35] S.V. Fedorov, *Nano-Structural Standard of Friction and Wear*, *Tribology in Industry*, vol. 40, no. 2, pp. 225-238, 2018.
- [36] I.V. Kragelskii, V.S. Schedrov, *Development of Science of Friction*, Moscow: USSR Academie Science, 1956. (in Russian)
- [37] G.A. Tomlinson, *A molecular theory of friction*, *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science*, vol. 7, iss. 46, 1929, doi: [10.1080/14786440608564819](https://doi.org/10.1080/14786440608564819)
- [38] S. Fedorov, *Nano-Structural Standard Of Wear*, in 15th International Conference on Tribology, 15-17 May, 2017, SERBIATRIB'17, Kragujevac, Serbia, pp. 526-535.
- [39] S.V. Fedorov, *The Calculation Of The True Volume Of Friction*, *Friction and lubrication in machines and mechanisms*, no. 12, pp. 3-7, 2010. (in Russian)