

# Mathematical Modeling of Journal Bearing Lubricated with Non-Newtonian Fluid

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## Keywords:

Journal hydrodynamic bearing  
Finite difference method  
Non-Newtonian lubricants  
Walther equation  
Reynolds equation

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Received: 22 May 2021

Revised: 7 July 2021

Accepted: 24 September 2021

## ABSTRACT

In this work it has been theoretically studied the influence of the non-Newtonian fluids on the lubricated hydrodynamic journal bearings. For this purpose, a FORTRAN program was developed to resolve the two-dimensional Reynolds equation in polar coordinates. Hence, a finite difference method was used. Initially, dynamic viscosity values are determined for the different non-Newtonian lubricants using Walther equation model. The journal bearing behavior has been simulated according to the eccentricity ratio, the  $L/D$ s ratio and the rheological characteristic index in case of industrial lubricating oils in accordance with ISO 3448 standard especially ISO-VG32, ISO-VG46 and ISO-VG68. The simulation has allowed performing analysis of journal bearing performances according to pressure, bearing load capacity and oil flow rate.

The study concludes that the hydrodynamic characteristics of the journal bearing are influenced by the type of lubricant, eccentricity ratio and the  $L/D$  ratio.

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## 1. INTRODUCTION

It is certain that mathematical modeling of lubrication problems requires a rheological model of the behavior of the lubricating fluid. Several studies have shown that the hypothesis of the Newtonian behavior of lubricants does not constitute a satisfactory approach in the sense of engineering.

The lubricants quality improvement is accompanied by the additives addition which allows on the one hand increasing the mechanical

devices performance and on the other hand to lead to the appearance of non-linear behavior which can significantly alter all the hydrodynamic characteristics such as pressure, load, torque and friction, etc. Lubricants exhibiting this non-linear character are qualified as non-Newtonian. There exist in the literature several constitutive laws, or equations of rheological behavior, which are used to model the non-Newtonian behavior of lubricants.

Hydrodynamic journal bearing is essential machine elements for compressors pumps, turbines, and internal combustion engines. In this

component the lubricant plays a very important role for their proper functioning and the choice of the right lubricant leads to the use of industrial fluids which have a non-linear behavior.

Several researchers have studied hydrodynamic bearings lubricated by non-Newtonian fluids, either theoretically or experimentally. In what follows have mentioned the most significant works.

Wada and Hayashi (1971 and 1974) [1,2], have studied experimentally and theoretically a plain journal bearing operating with pseudo-plastic lubricants, thus proving the applicability of their theoretical model. It was concluded that the pressure in the bearing, as well as the load capacity and frictional force are lower for pseudo-plastic lubricants, compared to a Newtonian lubricant, for the same operating conditions as the bearing.

Buckholz (1986) [3] performed theoretical analyzes on the hydrodynamic lubrication of finite journal bearing operating with power law model.

Li et al. (1996) [4] employed the finite volume method for the analysis of behavior operation of radial bearing operating with power law non-Newtonian lubricants. They also analyzed the influence of the roughness direction (transversal or longitudinal) in the film's tear contour.

Silva et al. (2001) [5] analyzed an infinitely large plain bearing, using lubricating oils with rheological characteristic index  $n > 1$  (dilatant lubricants) and  $n < 1$  (pseudo-plastic lubricants), as well as  $n = 1$  (Newtonian lubricants). The results are obtained by analytical resolution of the Reynolds equation in one-dimension. They showed that dilatants fluids improve load capacity and pressure.

A circular journal bearing has been investigated considering thermal effects by Singh and Majumdar (2005) [6]. The authors solved numerically the Reynolds equation simultaneously along with the energy equation and heat conduction equations to obtain the steady-state solution. the hydrodynamic characteristics are obtained for different L/D and eccentricity ratios.

Gertzos et al. (2008) [7] examined a journal bearing lubricated by a Bingham type fluid, through three-dimensional analysis by CFD, using the FLUENT software. The authors argued that the Reynolds boundary condition, although

more realistic, cannot be used, since FLUENT solves the Navier-Stokes equations instead of the Reynolds equation. Behavioral characteristics such as pressure distribution, specific eccentricity ratio, attitude angle, friction coefficient, lubricant flow and angle at which maximum pressure occurs were obtained not only for a Bingham fluid, but also for a Newtonian lubricant for various L / Ds ratios.

The effect of the polymer-containing oils and the base oil was investigated with experiment analysis for the case of the plain bearing designed with Babbitted and bronze by Kasai et al. (2012) [8]. The authors compared the friction coefficients for the both oils between the Babbitted bearing and the bronze bearing, those for the Babbitted bearing are placed at higher positions than for the bronze bearing.

A journal bearing lubricated by a Newtonian fluid mixed with some modified substances has been investigated by Lin et al. (2016) [9]. Some properties as dilatation and pseudo-plasticity have been analyzed. The authors concluded that bearing load capacity and friction loss are relatively higher in case of non-Newtonian lubricants. However, opposite conclusions are noted in case of pseudo-plastic fluids.

Chandra et al. (2016) [10] have analyzed the performance of a plain bearing with and without texturing lubricated with a non-Newtonian lubricant. The authors compared the characteristics of the both bearings and conclude that the bearing textured provides improved stability parameter than that of journal bearing without texturing.

A journal bearing lubricated by a non-Newtonian fluid has been investigated by Javorova et al. (2016) [11]. The authors choose the Rabinowitsch fluid model and was studied the effect of the elastic deformations and the behavior non-Newtonian of the lubricant on the steady state on the characteristics of a journal bearing with finite length.

In the current work, we present the effect of the behavior non-Newtonian fluids on the hydrodynamic characteristics of journal bearings. The modified Reynolds equation accounting for non-Newtonian fluid is solved numerically using the finite difference method.

A dynamic viscosity values are determined for the different non-Newtonian lubricants using the Walther equation model.

The variations of certain hydrodynamic characteristics such as pressure, load and flow rate are presented.

## 2. MATHEMATICAL FORMULATION OF THE PROBLEM

First, we present in this section the necessary equations which governing the hydrodynamic lubrication problem for the case of a journal bearing lubricated by a non-Newtonian fluid.

Next, the hydrodynamic characteristics of the bearings are examined. The lubricant fluid used in this study is considered a non-Newtonian fluid and the Walther relationship is applied to calculate the variation of viscosity with temperature.

### 2.1 Lubricant film thickness in bearing

The basic configuration of a hydrodynamic plain journal bearing in operation and some of its main physical and geometric parameters are presented in figure 1, where  $x$  is the circumferential coordinate,  $y$  is the radial coordinate and  $z$  is the axial coordinate. The expression of the thickness of the oil film can be determined from equation (Eq.1).

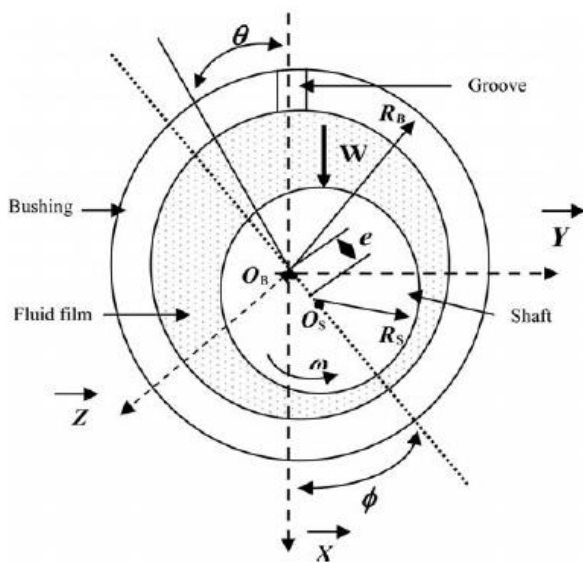


Fig. 1. Geometry of a plain journal bearing [12].

$$h(\theta) = c + e \cdot \cos \theta \quad (1)$$

### 2.2 Reynolds equation

In order to know the pressure field in the journal bearing, we have solved the Reynolds equation. This equation can be written for the case of a bearing lubricated by an incompressible fluid in steady state in the following form [13]:

$$\frac{\partial}{\partial x} \left[ \frac{h^{n+2}}{n} \frac{\partial P}{\partial x} \right] + \frac{\partial}{\partial z} \left[ h^{n+2} \frac{\partial P}{\partial z} \right] = 6\mu U^n \frac{\partial h}{\partial x} \quad (2)$$

Where  $n$  and  $U$  are the rheological characteristic index and the linear velocity of the rigid rotor respectively.

In order to obtain a more general solution, it is necessary to solve the Reynolds equation in its dimensionless form. For this, we use the following dimensionless parameters:

$$\begin{aligned} x &= R\theta; & \mu &= \mu_0 \bar{\mu} & z &= L\bar{z} \\ \bar{P} &= \frac{P}{2\pi\omega\mu_0 \left(\frac{R}{h_0}\right)^2 \left(\frac{U}{C}\right)^{n-1}} & h &= CH \end{aligned} \quad (3)$$

Substituting equation (Eq. 3) into equation (Eq. 2), we obtain the Reynolds equation in non-dimensional form (Eq. 4).

The pressure field in the journal bearing is given by the resolution of the Eq.4.

$$\frac{\partial}{\partial \theta} \left[ \frac{H^{n+2}}{n\bar{\mu}} \frac{\partial \bar{P}}{\partial \theta} \right] + \left(\frac{R}{L}\right)^2 \frac{\partial}{\partial \bar{z}} \left[ \frac{H^{n+2}}{\bar{\mu}} \frac{\partial \bar{P}}{\partial \bar{z}} \right] = 6 \left(\frac{h_0}{C}\right)^2 \frac{\partial H}{\partial \theta} \quad (4)$$

The boundary conditions associated with the pressure distribution are those of Sommerfeld. That is, the pressure equal to zero at the ends of the both directions axial and circumferential.

### 2.3 Performance characteristics

The load-carrying capacity  $W$  supported by the plain bearing is determined by integration of the field of the pressure around the journal. This load capacity can be broken down into a component parallel to the median line and a component perpendicular to this line. These two components are called radial force and tangential force, respectively. Therefore, we can write:

$$\bar{W}_r = \bar{W} \cdot \cos \theta = \int_0^1 \int_0^{2\pi} \bar{P} R \cos \theta d\theta d\bar{z} \quad (5)$$

$$\bar{W}_r = \bar{W} \cdot \sin \theta = \int_0^1 \int_0^{2\pi} \bar{P} R \sin \theta d\theta dz \quad (6)$$

Therefore, the dimensionless load capacity  $\bar{W}$  of the bearing can then be calculated by:

$$\bar{W} = \sqrt{\bar{W}_r^2 + \bar{W}_t^2} \quad (7)$$

The circumferential dimensionless flow at the inlet of the lubricating oil wedge,  $\bar{Q}$ , at the angular position  $\theta = 0$ , can then be calculated by the equation (Eq. 8):

$$\bar{Q} = \left( \pi H - \frac{\pi H^{n+2}}{6n\mu} \int_0^1 \frac{\partial \bar{P}}{\partial \theta} dz \right)_{\theta=0} \quad (8)$$

### 3. VISCOSITY-TEMPERATURE EQUATION

It is clear that for lubricating oil, viscosity point out its resistance to motion. It is varying according to several parameters as temperature, pressure and shear rate.

Whatever the field of application, mineral and synthetic oil show a linear relation between the shear stress and the shear rate. For this reason, they are called Newtonian fluids. In case of journal bearings hydrodynamic analysis, it is assumed that the viscosity is a function of the temperature only, which is an acceptable assumption for regular mineral and synthetic oils.

The hydrodynamic lubrication regime is characterized by the high sensitivity of lubrication oil to the operating temperature. The later determines the lubricant film thickness between the journal bearing walls. Equations describing relation between viscosity and temperature are very numerous. Some of them are empirical whereas others are obtained from theoretical model. Furthermore, Viscosity-temperature ASTM chart are also used.

There exist various formulas to obtain the viscosity a lubricant from the know values in the pure regimes at a given temperature.

The viscosity variation as a function of temperature is obtained by the McCoull and Walther relationship [14].

$$\log(\log(\nu + 0.6)) = a \log(T) + b \quad (9)$$

In the above equation,  $\nu$  is the kinematic viscosity of the lubricant and  $a$  and  $b$ , being two constant values determined of the Kinematic viscosity of the lubricant mainly in the range of  $40^\circ\text{C}$  to  $100^\circ\text{C}$  table1. from the equations below:

$$a = \frac{\log(\log(\nu_1 + 0.6)/\log(\nu_2 + 0.6))}{\log(T_1/T_2)} \quad (10)$$

$$b = \log(\log(\nu_1 + 0.6)) - a \cdot \log(T_1) \quad (11)$$

$\nu_1$  and  $\nu_2$  are kinematic viscosity of the lubricant at temperatures  $T_1$  and  $T_2$ .

**Table 1.** Performances indices for different lubricants.

Parameters	Lubricants		
	ISO-VG 32	ISO-VG 46	ISO-VG 86
Kinematic Viscosity at $T_1 = 40^\circ\text{C}$ $\nu_1$ [Cst]	28	38	60
Kinematic Viscosity at $T_2 = 100^\circ\text{C}$ $\nu_2$ [Cst]	4.2	5.5	7

### 4. NUMERICAL SOLUTION

Eq. 4. is the Reynolds equation for hydrodynamic lubrication for non-Newtonian fluids in the dimensionless form. In this work, the finite difference method is used to discretized Eq. 4. and the system of algebraic equations is solved using the iterative Gauss-Seidel method with over-relaxation scheme.

The dimensional and operating characteristics of the plain journal bearing are explained in the table below:

**Table 2.** dimensional and operating characteristics of the plain journal bearing.

Parameters	Values
Journal radius, R (mm)	50
Bearing length, L(mm)	25 - 50 - 100
Radial clearance, C(μ m)	0.05
Rotational speed, ω (rpm)	1500
Lubricant density, (Kg/m <sup>3</sup> )	809
Dynamic viscosity, μ (Pa.s)	Walther equation
Eccentricity ratio, ε (-)	0.1 - 0.9
Temperature, T <sub>0</sub> , (°C)	35

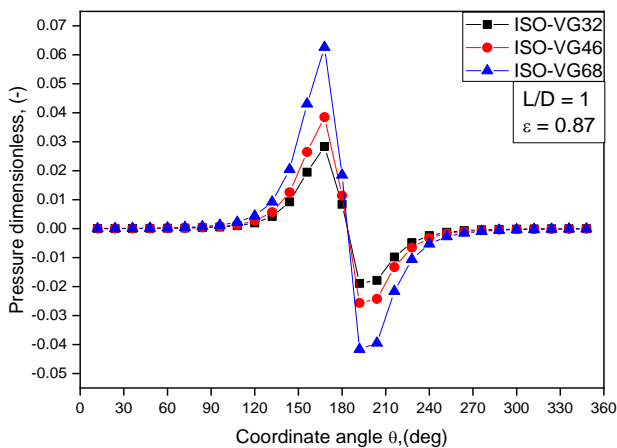
In this study, an adequate number of mesh points are chosen such that  $K_Z=21$  points in the axial direction and  $K_X=31$  points in the circumferential direction.

After initializing the variables of the problem, we solve the Reynolds equation. Convergence on the pressure is reached when the relative difference in pressure between two successive iterations is less than  $10^{-5}$ .

### 5. RESULTS AND DISCUSSION

Fig. 2 shows the dimensionless pressure distribution along angular coordination in the median plane position of the bearing.

The results are related to different lubricants for  $L/D=1$  and  $\varepsilon=0.87$ . The figure shows the existence of a maximum pressure zone between  $150^\circ$  and  $160^\circ$  for the three lubricants. It should be noted that the lubricant ISO VG 68 presented a maximum value.

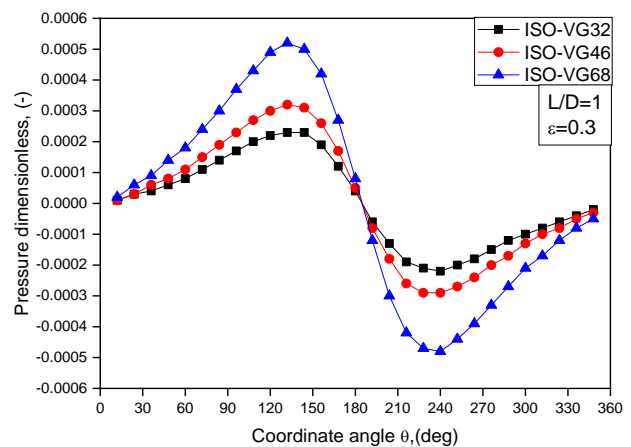


**Fig. 2.** Dimensionless pressure distribution in the circumferential direction for three lubricants with  $\varepsilon=0.87$  and  $L/D=1$ .

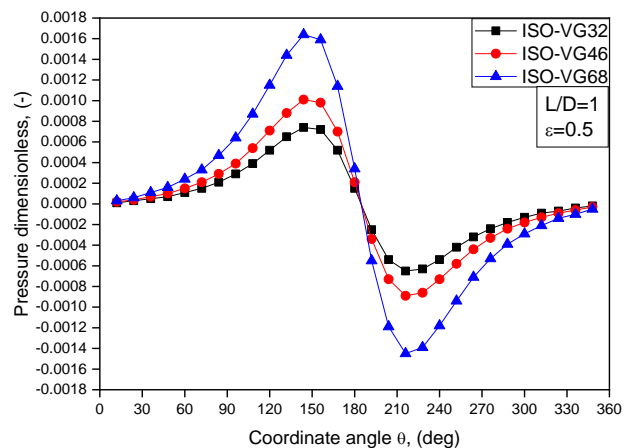
Figs 3, 4, 5, and 6 presents the effect of different values of eccentricity ratio  $\varepsilon$  on the circumferential distribution of dimensionless pressure in the median plane position of the bearing.

The results are related to different values of eccentricity ratio  $\varepsilon$  such as (0.3, 0.5, 0.7 and 0.9) and  $L/D=1$ . The figures show the existence of a zone of maximum pressure varies as a function of the eccentricity ratio, this zone is concentrated in the middle of the bearing and corresponding to an angle between approximately  $120^\circ$  and  $180^\circ$  for an eccentricity ratio equal to 0.3, and  $80^\circ$  and  $120^\circ$  for an eccentricity ratio equal to 0.9. It should be noted that the increase in the eccentricity ratio  $\varepsilon$ , generates

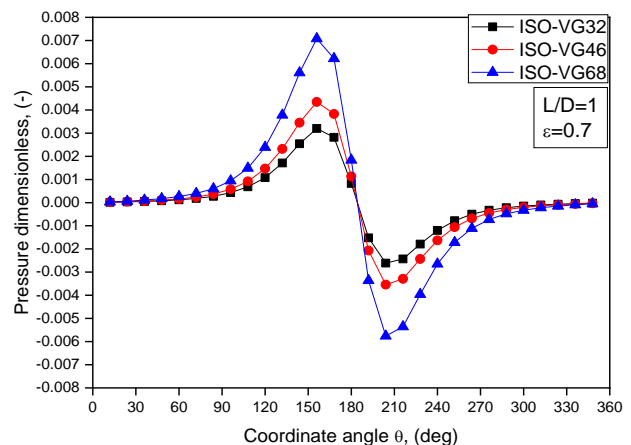
a considerable hydrodynamic pressure. The comparison between Figs. 3, 4, 5 and 6, highlights the proportionally character between the pressure peak and eccentricity ratio  $\varepsilon$  for the same lubricant.



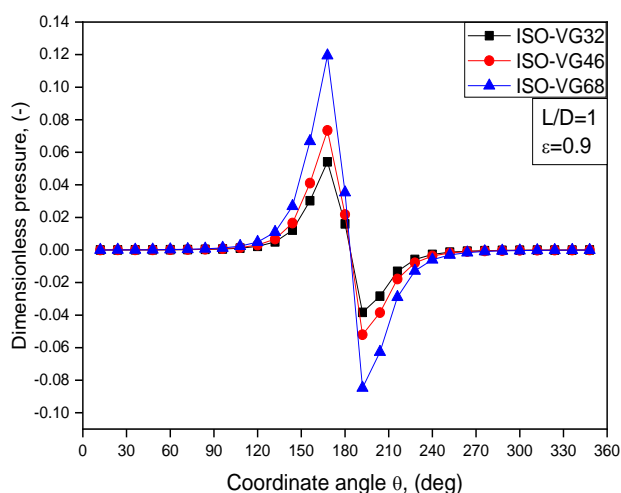
**Fig. 3.** Dimensionless pressure distribution in the circumferential direction for three lubricants with  $\varepsilon=0.3$  and  $L/D=1$ .



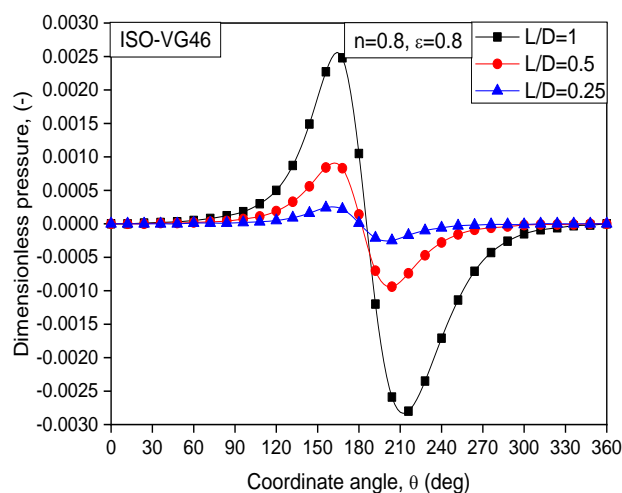
**Fig. 4.** Dimensionless pressure distribution in the circumferential direction for three lubricants with  $\varepsilon=0.5$  and  $L/D=1$ .



**Fig. 5.** Dimensionless pressure distribution in the circumferential direction for three lubricants with  $\varepsilon=0.7$  and  $L/D=1$ .

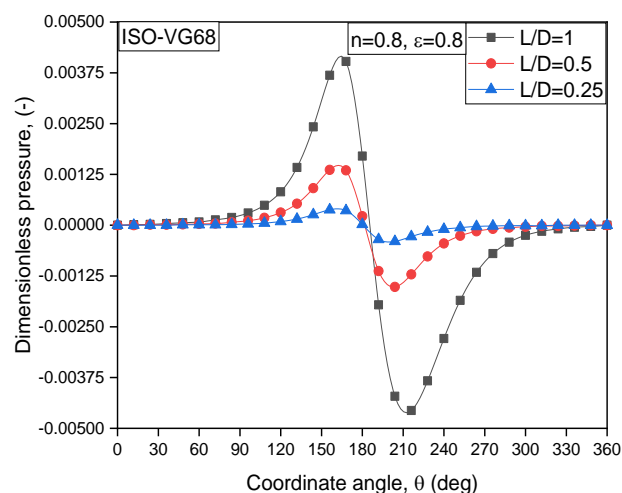


**Fig. 6.** Dimensionless pressure distribution in the circumferential direction for three lubricants with  $\varepsilon=0.9$  and  $L/D=1$ .

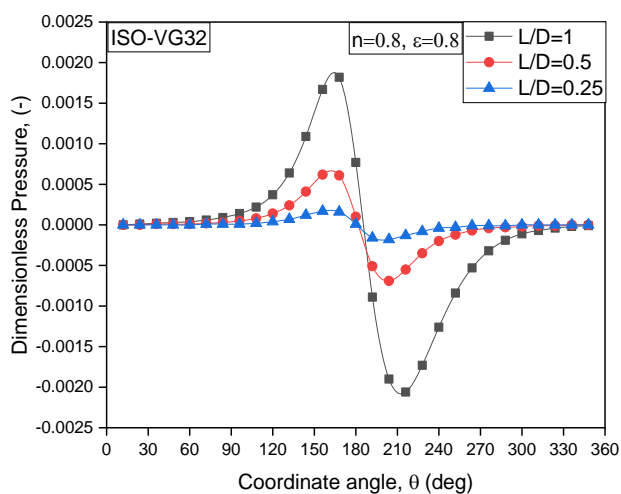


**Fig. 8.** Dimensionless pressure distribution in the circumferential direction for three values of  $L/D$ s ratio with  $\varepsilon=0.8$  and  $n=0.8$ , for the lubricant *ISO-VG46*.

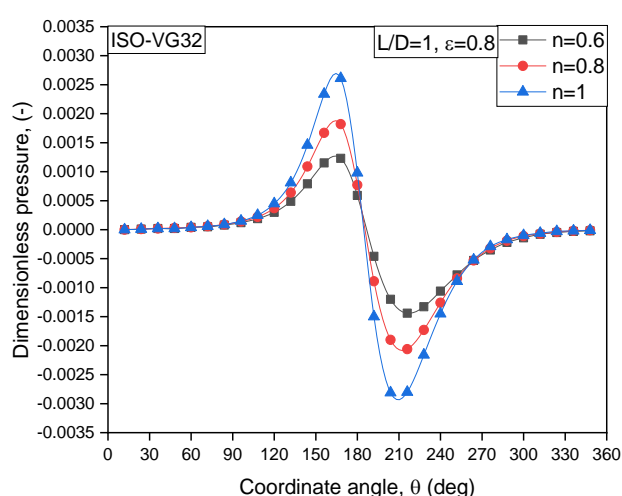
Figs. 7, 8 and 9 shows the bearing dimensionless pressure distribution vs coordinate angle of the lubricants (*ISO-VG32*, *ISO-VG46* and *ISO-VG68*) for an eccentricity ratio and rheological characteristic index equals to  $0.8$  and  $0.8$  respectively. The results are for three values of  $L/D$ s ratio such as ( $1$ ,  $0.5$  and  $0.25$ ). The pressure in bearing for the lubricant *ISO-VG68* for different values of  $L/D$ s ratio was higher than those for the others lubricants at the middle bearing angles. However, the results for the same plain bearing show that the pressure data of the bearing lubricated with *ISO-VG68* is maximized and that the pressure peaks are close to that of the bearings lubricated with *ISO-VG 32* and *ISO-VG 46*.



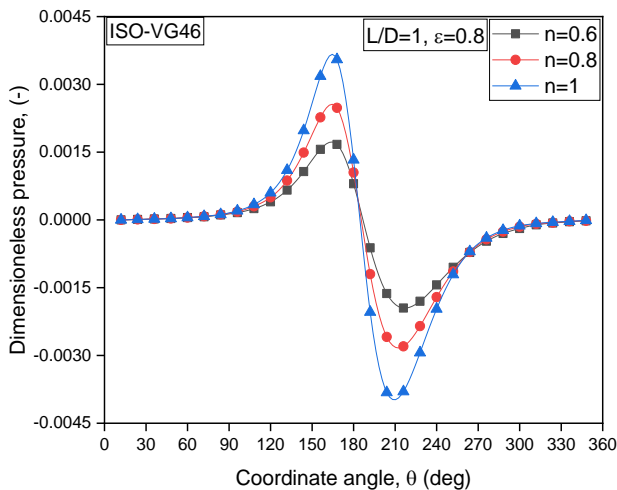
**Fig. 9.** Dimensionless pressure distribution in the circumferential direction for three values of  $L/D$ s ratio with  $\varepsilon=0.8$ ,  $n=0.8$ , for the lubricant *ISO-VG68*.



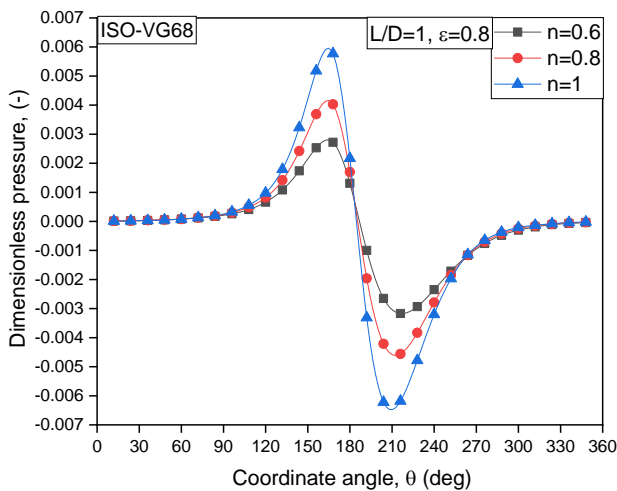
**Fig. 7.** Dimensionless pressure distribution in the circumferential direction for three values of  $L/D$ s ratio with  $\varepsilon=0.8$ ,  $n=0.8$ , for the lubricant *ISO-VG32*.



**Fig. 10.** Dimensionless pressure distribution in the circumferential direction for three values of rheological characteristic index for the lubricant: *ISO-VG32* with  $\varepsilon=0.8$  and  $L/D=1$ .



**Fig. 11.** Dimensionless pressure distribution in the circumferential direction for three values of rheological characteristic index for the lubricant: *ISO-VG46* with  $\varepsilon = 0.8$  and  $L/D = 1$ .

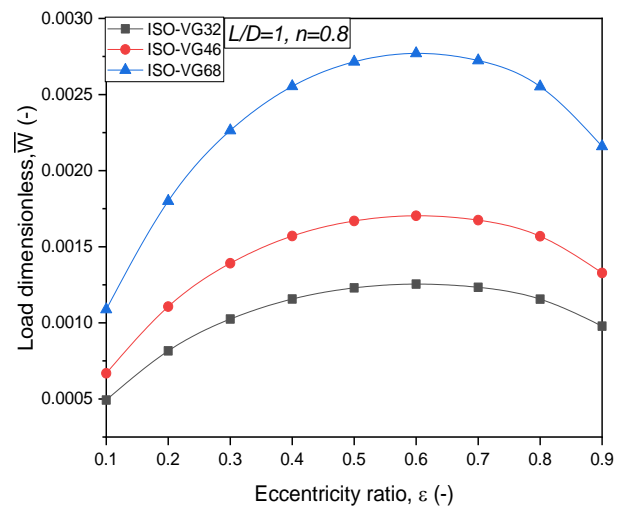


**Fig. 12.** Dimensionless pressure distribution in the circumferential direction for three values of rheological characteristic index for the lubricant: *ISO-VG68* with  $\varepsilon = 0.8$  and  $L/D = 1$ .

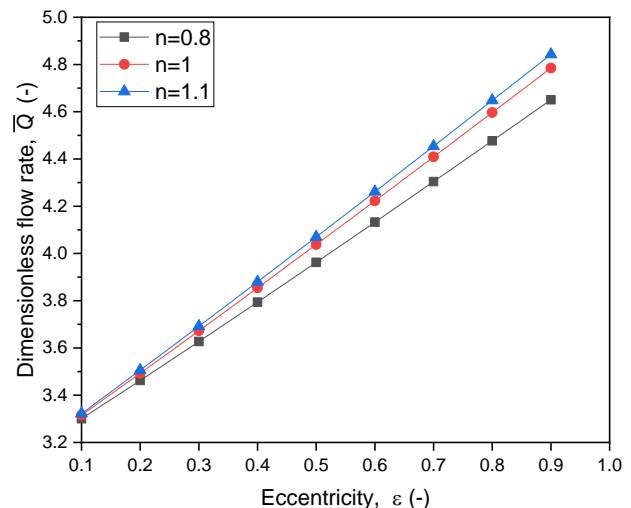
Figs. 10, 11 and 12 shows the influence of the rheological characteristic index on the dimensionless pressure variation for a shaft rotation speed equal to 1500 rpm. The comparison between these figures shows that the increase in the value of the rheological characteristic index shows a slight increase in the values of the hydrodynamic pressure and the peaks of the pressures are close.

Figure 13 shows the dimensionless load carrying capacity  $\bar{W}$  as a function of the eccentricity ratio  $\varepsilon$  for different lubricants (*ISO-VG32*, *ISO-VG46* and *ISO-VG68*). It is observed that the values of the load capacity  $\bar{W}$  increase

with increasing value eccentricity ratio pass by a maximum equal to 0.6, and then decreases and this for the three lubricants.



**Fig. 13.** Evolution of dimensionless load versus eccentricity ratio for three lubricants with  $\omega = 1500$  rpm,  $L/D = 1$  and  $n = 0.8$ .



**Fig. 14.** Dimensionless inlet circumferential flow rate,  $\bar{Q}$  versus eccentricity ratio for three values of rheological characteristic index for  $L/D = 1$ . for the lubricant *ISO-VG68*.

Fig. 14 shows the dimensionless inlet flow rate as a function of the eccentricity ratio for three different values of the rheological characteristic index  $n$ , (0.8, 1 and 1.1) and  $L/D = 1.0$  for the lubricant *ISO-VG68*. It is observed that the dimensionless inlet flow,  $\bar{Q}$  increases with increasing eccentricity ratio the behavior is similar and there is not much variation of dimensionless flow values when the eccentricity ratio is less than 0.5.

## 6. CONCLUSION

In this paper, the effect of the behaviour non-Newtonian fluids on the hydrodynamic characteristics of a journal bearing was investigated.

In order to achieve this, three different lubricants have been restrained to carry out the hydrodynamic simulation. These are the following lubricants: ISO-VG32, ISO-VG46 and ISO-VG68. It is important to point out that a significant increase in the pressure in the bearing was induced by the  $L/D$  ratio increase.

The journal bearing simulation has allowed to conclude that the increase in the eccentricity ratio results in an increase in the pressure in the bearing and in the dimensionless load capacity while the increase in the lubricant's rheological characteristic index results from the increase in pressure in a proportional manner.

Another finding is that hydrodynamic bearings operating with an ISO-VG 32 type lubricant has a lower load capacity. On the other hand, the ISO-VG 68 type lubricant gives a greater load capacity.

The rheological characteristic index of lubricating oil also has a great influence on the hydrodynamic characteristics of the bearing. The variation of the pressure in the lubricating film is much more accentuated for high values of the rheological characteristic index for the same lubricant.

Finally, the influence of the behaviour of non Newtonian lubricants cannot be neglected, seeing that the viscosity influences the hydrodynamic characteristics of lubricated bearings.

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## NOMENCLATURE

Name	Designation	Units
$C$	Clearance	(m)
$e$	Eccentricity	(m)
$h$	Fluid film thickness	(m)
$H$	Film thickness dimensionless	(-)
$L$	Bearing length	(m)
$P$	Pressure of the lubricant	(Pa)
$n$	Rheological characteristic index	(-)
$\mu$	Lubricant dynamic viscosity	(Pa.s)
$U$	Journal velocity, parallel to the film	(m/s)
$\mu_0$	Viscosity dynamic at $T_0$	(Pa.s)
$\bar{\mu}$	Dimensionless dynamic viscosity	(-)
$\rho$	Lubricant density	(kg/m <sup>3</sup> )
$\bar{P}$	Dimensionless pressure	(-)
$\bar{W}$	Dimensionless load carrying	(N)
$x, z$	Cartesians coordinate	(-)
$\varepsilon$	Eccentricity ratio $\varepsilon=e/C$	(-)
$\omega$	Rotational speed of journal	(rpm)
$\theta$	Angular coordinate of the bearing	(°)
K_X	Nodes along the circumferential direction	(-)
K_Z	Nodes in the axial direction.	(-)